Selection in Information Acquisition and Monetary Non-Neutrality

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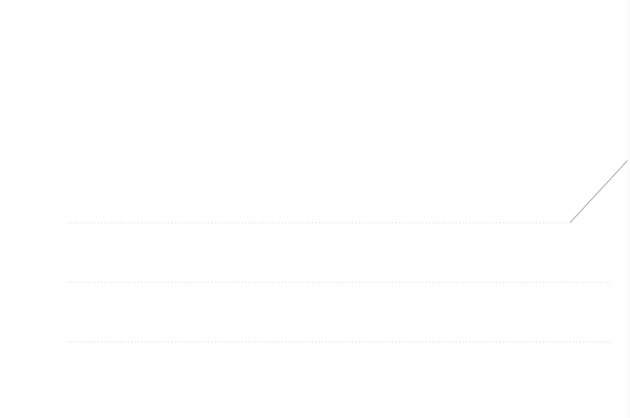
Motivation

- The average firm is highly uncertain about economic outcomes.
- But there is a high degree of heterogeneity in subjective uncertainty.
- **This Paper:** Whose expectations matter for macroeconomic outcomes?
- **Summary:**
    - Subjective uncertainty is *positively* correlated w/ time since last price change (***selection***)
    - A model with ***state-dependent information acquisition*** explains this selection
    - Only ***the most informed*** firms' expectations matter for output response

Motivation

Subjective uncertainty: standard deviation of belief about desired price change



There is a lot of heterogeneity in uncertainty across firms.

Motivation

Firms that changed their prices more recently have more accurate expectations.

	(1)	(2)	(3)	(4)
Dependent variable: Subjective uncertainty about firms' desired price changes				
Dummy for price changes (last 12 months)	-0.112* (0.057)	-0.210*** (0.063)	-0.265*** (0.056)	
Time elapsed since price change				0.010* (0.005)
Observations	485	488	486	487
R-squared	0.061	0.170	0.243	0.188
Industry fixed effects	Yes	Yes	Yes	Yes
Firm-level controls		Yes	Yes	Yes
Manager controls			Yes	Yes

Model: Rational Inattention + Calvo

Model: Firms, Shocks and Payoffs.

- Time is continuous and indexed by $t \geq 0$.
- There is a measure of price-setting firms indexed by $i \in [0, 1]$.
- i's instantaneous profit:

$$\bar{\Pi} - B(p_{i,t} - p^*_{i,t})^2$$

- Each firm follows an exogenous *desired* price:

$$\mathrm{d}p^*_{i,t} = \sigma \mathrm{d}W_{i,t}$$

- Price change opportunities arrive at Poisson rate θ (Calvo).

Model: Information Structure and Cost of Attention.

- Firm i does not observe $p^*_{i,t}$ but see a signal process over time:

$$\mathrm{d}s_{i,t} = p^*_{i,t}\mathrm{d}t + \sigma_{s,i,t}\mathrm{d}W_{s,i,t}$$

- Information sets:

$$S_{i,t} = \{s_{i,\tau} : 0 \leq \tau \leq t\} \cup S_{i,0}, \;\; S_{i,0} \text{ given.}$$

- Attention problem: firm chooses $\sigma_{s,i,t} \in \mathbb{R}_+ \cup \{\infty\}$ for all $t \geq 0$.

- Cost of information increases with rate of reduction in differential entropy

$$C(\mathrm{d}\mathbb{I}(P^*_{i,t}; S_{i,t})) : \quad C'(.) \geq 0, \quad \mathbb{I}(P^*_{i,t}; S_{i,t}) \equiv h(P^*_{i,t}|S_{i,0}) - h(P^*_{i,t}|S_{i,t})$$

Model

$$\min_{\{\sigma_{s,i,t}\geq 0, \tilde{p}_{i,t}: t\geq 0\}} \mathbb{E}[\int_0^\infty e^{-\rho t}[\underbrace{B(p_{i,t}-p^*_{i,t})^2 \mathrm{d}t}_{\text{loss from mis-pricing}} + \underbrace{C(\mathrm{d}\mathbb{I}(P^*_{i,t}; S_{i,t}))}_{\text{cost of information}}]|S_{i,0}]$$

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$$s.t.\ \mathrm{d}p_{i,t} = (\tilde{p}_{i,t}-p_{i,t})\mathrm{d}\chi_{i,t},\ \chi_{i,t}\sim \text{Poisson}(\theta)$$

$$\mathrm{d}s_{i,t} = p^*_{i,t}\mathrm{d}t + \sigma_{s,i,t}\mathrm{d}W_{s,i,t},\ S_{i,0}, p_{i,0} \text{ given.}$$

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Today, two extremes of convexity for $C(\mathrm{d}\mathbb{I})$:

- Linear: $C_L(\mathrm{d}\mathbb{I}) = \omega \mathrm{d}\mathbb{I}$
- Extremely Convex: $C_F(\mathrm{d}\mathbb{I}) = \begin{cases} 0 & \mathrm{d}\mathbb{I} \leq \bar{\lambda}\mathrm{d}t \\ \infty & \mathrm{d}\mathbb{I} > \bar{\lambda}\mathrm{d}t \end{cases}$

Mapping Model Objects to the Data

Definition
We define firm i's ***true price gap*** *and* ***perceived price gap****, and* ***subjective uncertainty*** *as*

$$x^*_{i,t} \equiv p^*_{i,t} - p_{i,t}, \quad x_{i,t} \equiv \mathbb{E}[x^*_{i,t}|S_{i,t}], \quad z_{i,t} \equiv \mathbb{V}ar(x^*_{i,t}|S_{i,t})$$

respectively.

State variables for firm's problem: (belief distribution about $x^*_{i,t}$)

- $x_{i,t}$: how much firm ***thinks*** its price is from optimal price
- $z_{i,t}$: subjective uncertainty

Results

Results

Theorem (Optimal Information Acquisition with Linear Cost)

1. *It is optimal for firms to never acquire information in between price changes, and uncertainty grows linearly with time.*
2. *Upon the arrival of an opportunity for a price change, firm acquires enough information to reset their uncertainty to Z^* where*

$$\frac{1}{Z^*} = \frac{B}{\omega(\rho+\theta)} + \theta \int_0^\infty e^{-(\rho+\theta)h} \frac{1}{Z^* + \sigma^2 h} dh \tag{1}$$

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Proposition (Optimal Information Acquisition with Convex Cost)

All firms have the same uncertainty, independent of their state:

$$z = \frac{\sigma^2}{\bar{\lambda}} \tag{2}$$

Aggregation

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- *with the linear cost is an exponential with rate* θ/σ^2 *shifted by* Z^*.

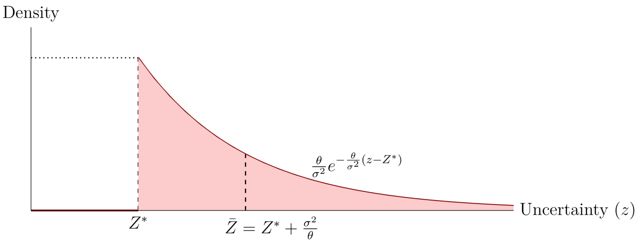

Figure I: Distribution of Uncertainty Across Firms

Implications for Monetary Non-Neutrality

Monetary Non-Neutrality

- Consider a permanent shock to $x^*_{i,0}$ of size δ, and define

$$M(x,z,\delta) = \int_0^\infty \mathbb{E}_0 \left[y_{i,t} | x^*_{i,0} = x + \delta, z_{i,t} = z \right] \mathrm{d}t, \quad \mathcal{M}(\delta) = \int M(x,z,\delta) \tilde{F}(\mathrm{d}x, \mathrm{d}z)$$

Theorem (*Sufficient statistic with linear cost*)
Cumulative response of output to a 1 percent monetary shock (area under IRF):

$$\mathcal{M}(1) = \underbrace{\frac{1}{\theta}}_{\text{inverse frequency of price change}} + \underbrace{\frac{Z^*}{\sigma^2}}_{\text{subjective (normalized) uncertainty of price-setters}} \tag{3}$$

- Main takeaway:
  Only the most informed firms' expectations matter for monetary non-neutrality

Conclusion

- Evidence suggests there is selection in information acquisition.
- This is consistent with a state-dependent information acquisition model.
- Selection implies that only the most informed firms' expectations matter for output response to monetary shocks.

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- Optimality of $\lambda_{i,t}$ implies $var(\Delta p_{i,t}) = \sigma^2 h$.

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- So $\Delta p_{i,t}$ is generated by a Brownian motion of scale σ.
- In hypothetical economy assign $\Delta p_{i,t}$ to a firm whose ideal price is $p_{i,t}$.
- The hypothetical economy is as if it has no information frictions but has the same distribution of price changes.

Monetary Non-Neutrality

- Because it takes time for firms to become aware of the shock when it is unannounced:

$$db = -\lambda(z)b + U,$$
$$\lambda(z) = 1 - \frac{Z^*}{z}$$

- In fact:

$$\mathcal{M}(F_b) - \mathcal{M}(F_x) = \frac{Z^*}{\sigma^2}$$

- Need to know uncertainty conditional on price change.