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# INDETERMINACY OF RATIONAL EXPECTATIONS EQUILIBRIA IN SEQUENTIAL FINANCIAL MARKETS

BY PAOLA DONATI

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# INDETERMINACY OF RATIONAL EXPECTATIONS EQUILIBRIA IN SEQUENTIAL FINANCIAL MARKETS[1]

BY PAOLA DONATI[2]

September 2003

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## Contents

**Abstract**

We provide a general characterization of the structure of rational expectations equilibria of any degree of revelation for pure exchange, sequential economies, with finitely many states of private information, an incomplete financial market and nominal assets. We estimate the dimension of the rational expectations equilibria for any degree of revelation. Then, we show how a central bank, by deciding on the money supply, may affect the revelation of information at equilibrium.

*Key words*: Incomplete markets, Indeterminacy; Information revelation; Monetary Policy.
*JEL classification numbers*: D52, D80, D82, E52.

# Non Technical Summary

We consider a standard, sequential, perfectly competitive, economy under uncertainty, with an incomplete financial market, nominal assets, and asymmetric information.
Financial markets are incomplete when individuals do not have access to as many assets as they would require to get fully insured against any risk and uncertainty they face. Assets are nominal when have payoffs denominated in abstract unit of accounts, that is they payoff in terms of money.

In the first period, finitely many heterogeneous individuals have access to finitely many sources of private information on future economic events, and take their consumption and savings decisions conditionally. Hence, first period equilibrium prices may pool and reveal the economy's private information.

We assume that individuals have rational expectations, which means that they correctly establish the relationship between equilibrium prices and exogenous economic variables, and that they know the map from the aggregate information states to equilibrium prices. These assumptions imply that individuals may refine their expectations by learning from the prices they observe on the market.

However, when the financial market is incomplete and nominal inside assets are exchanged, competitive equilibria display a (real) indeterminacy of equilibrium prices and allocations. Such indeterminacy persists when individuals are asymmetrically informed, but its dimension changes.
The first contribution of this paper is to provide a general characterization of the dimension of the indeterminacy of rational expectations competitive equilibria under asymmetric information.
By parametrizing the space of economies in terms of utilities, we establish the generic existence of fully revealing and of partially revealing rational expectations equilibria and we show that rational expectations equilibria of any information structure derived from prices satisfy the property of regularity necessary to comparative statics analysis.
By parametrizing the space of economies in endowments, we provide a general characterization of the dimension of the set of fully non revealing and of partially revealing rational expectations equilibria.

Next, starting from the observation that such degrees of indeterminacy can be exploited to fix prices in a way that affects the revelation of information at equilibrium, we show how a central bank, by deciding on the money supply, and thereby affecting the price levels, influences the allocation of resources at equilibrium in two ways: first, by parametrizing the (real) indeterminacy due to the incompleteness of the nominal asset market, and, second, by determining the information prices convey.

The second contribution of this paper is therefore to propose algorithms that allows to determine the money supply needed to determine the full, partial, or null degree of information disclosure by rational expectations equilibrium prices.

Finally, in the framework of an example and by means of numerical simulations, we study the normative properties of rational expectations equilibria according to their level of information disclosure. The assumption is that the central bank implements the monetary policy that maximizes the value of an expected social welfare function. Heterogeneity matters: different individuals are differently affected by the degree of information revelation by rational expectations equilibrium prices.

# 1 Introduction

In intertemporal economies under uncertainty and asymmetric information, individuals with rational expectations formulate forecasts on *tomorrow's* equilibrium outcomes after refining their initial beliefs with the information they extract from the prices they observe on the market *today.*

The rational expectations hypothesis requires in fact that individuals correctly establish the relationship between the equilibrium price domain and the exogenous economic variables, and that they exploit their knowledge of the price map to perfect their private information. As a consequence, if an exhaustive specification of the fundamentals of the economy corresponds to each state of aggregate private information, and today observed prices convey such joint signal entirely, that is if prices are *fully revealing,* individuals with rational expectations may foresee the equilibrium value of tomorrow's variables exactly. If rational expectations equilibrium prices disclose just some, but not all, of the individual private signals, that is if they are *partially revealing*, or if they do not provide any (private) informational feedback at all, that is if they are *fully non revealing,* individuals are prevented from completing their knowledge refinement and remain asymmetrically informed at equilibrium.

In sequential general equilibrium economies under uncertainty, the revelation or non-revelation content of rational expectation equilibrium prices depends on the asset structure.
Radner [22] first showed that in a standard two-period general equilibrium economy where asymmetrically informed individuals receive a *finite* number of private signals, exchange *real assets* in an *incomplete financial market* and the equilibrium price vector has *higher dimensionality* than the space of individual private information, rational expectation equilibrium prices are typically fully revealing. Pietra and Siconolfi [19] and Stahn [24] extended Radner's result and establish the existence of fully revealing rational expectations equilibria for intertemporal economies in which the physical commodities at each spot are strictly more than one. This implies that the budget constraints individuals face when have to decide on their income allocation across time cannot be trivially reduced to one. In such 'nontrivial' sequential economies, in fact, the number of budget constraints in an optimization problem is determined by the level of refinement of individual knowledge. This is defined by the *join* of the partitions induced on the state space both, by private signals and, by the information conveyed by equilibrium prices. The finer the join partition, the finer individual's knowledge and the lower the number of budget constraints in his optimization problem.

When individuals transfer their revenue across time and states of the world by means of *nominal assets*, that is by means of assets paying off in terms of unit of account, as opposed to the real asset setting of Radner's, the indeterminacy of equilibrium[1] can be exploited to fix the degree of information disclosure by

[1] See Cass [5], Balasko and Cass [2], Geanakoploas and Mas-Colell [8].

rational expectation equilibrium prices. In particular, Rahi [23] proved the existence of partially revealing rational expectations equilibria by imposing ad hoc assumptions on individuals' information at equilibrium: in order to clear the markets in the presence of asymmetric information at equilibrium, he needed to allow that some individual both is not constrained by the incompleteness of the asset market – a trick suggested by Cass [4] and Magill and Shafer [14] to guarantee that aggregate demand be always well-behaved – and that he is endowed with the pooled information of all market participants. Mischel, Polemarchakis and Siconolfi [16] and Polemarchakis and Siconolfi [21] showed that fully non revealing rational expectations equilibria exist if individual first period endowments and, importantly, preferences over first period consumption bundles, are private information invariant – restrictions these that are crucial to prevent information disclosure through today prices –. Citanna and Villanacci [6], borrowing Polemarchakis and Siconolfi's [21] notion of equilibrium and the economic rationale for the existence of fully non revealing rational expectations equilibria, proved the existence of rational expectations equilibria for any informational structure derived from prices by means of homotopy methods.

The first part of this paper centers on the structure of the set of equilibrium outcomes of nominal asset, nontrivial, sequential economies with finitely many states of private information. In particular, it addresses the question of the dimension of the set of rational expectations equilibria as a function of the amount of information disclosed by prices. The approach is minimal, under the assumptions Polemarchakis and Siconolfi [21] needed to guarantee non revelation, we show that, without loss of generality, one single nominal asset suffices to characterize the generic *dimension of the set of equilibrium allocations* for any degree of information disclosure.

The signal invariance of first period endowments and preferences required by Polemarchakis and Siconolfi [21], imposes a twist from standard perturbation techniques. To prove the generic existence of fully revealing equilibria, proof that is required to demonstrate the regularity of fully informative economies, we parametrize the space of economies in terms of utility functions. Then we show that for any regular, normalized economy, the set of fully revealing rational expectations equilibria is typically finite and zero dimensional.

Furthermore, by an ad hoc parametrization of the space of economies in terms of endowments, we establish the generic regularity of fully and partially noninformative economies. Afterward, by a transversality argument, we demonstrate that the sets of rational expectations equilibria of these noninformative economies typically contains submanifolds of strictly positive dimensions.
The importance of the finiteness of the equilibrium set was originally pointed out by Debreu [7]. Since this property is absent in nominal asset economies with asymmetric information, an explicit modelling of monetary policy action is required. This is the aim of the second part of the paper.

Since in sequential economies with asymmetric information and an incomplete nominal asset market the degree of private information disclosure can be seen in terms of equilibrium selection, in the second part of this paper we show how a central bank may actually decide on the revelation of information at a rational expectations equilibrium.

The initial economy set up is enriched by assuming that individuals use paper *money* as a medium of exchange and employ nominal assets, in particular bonds with a state independent return, as a store of value. There is *no exogenous money supply* and to explain its creation (and its destruction) we refer to the model originally proposed by Wicksell [27] where money is produced by a central bank that acts as a pure intermediary. At the beginning of each time period, individuals sell the central monetary authority securities to get in exchange the money they need for transaction purposes. The creation of money is due to the fact that the bank buys assets which are not a medium of payment and pays them back with money which is a means of payment. At the end of every time period, individuals' money balances are cleared, so that all the liquidity injected in the economy goes back to the central bank. This is explained by the fact that to make income transfers across time individuals use bonds which, as opposed to money, yield a strictly positive return. Via such *intra-period open market operations* on the securities issued by individuals in exchange for money, the central bank fixes the monetary value of the volume of trade thereby (indirectly) it affects price levels. Thus, the central bank also influences the information prices convey.

We show *how* monetary policy may affect the revelation of information at a rational expectations equilibrium by means of different *algorithms* that allow to select the monetary policy plans yielding fully revealing, fully non revealing or partially revealing equilibria.

In general, in economies with an incomplete asset market where the financial structure is taken as given, competitive equilibria not only are typically ex - ante inefficient, that is they are Pareto suboptimal, but the available assets are also inefficiently used (Geanakoplos and Polemarchakis [9]). Thus, in general, under these assumptions the market by itself does not make an optimal use of the available resources. Weiss [26] showed that when heterogeneous individuals with rational expectations have access to private sources of information, monetary policy is effective when both it is fully anticipated and it ensures that market prices be consistent with all the relevant information of the economy because. This is because in such a way monetary policy indirectly communicates which factors are relevant for an efficient allocation of resources. Weiss' thesis relies on also Tobin [25], who demonstrated that when in conditions of uncertainty individuals exhibit a preference for liquidity, monetary policy, having an impact on the payoffs of the traded assets, may affect individual decisions on the allocation of income between consumption and investment. However, as pointed out by Hirshleifer [12], information may be another source of welfare loss. Prices may in fact transmit too much information to the economy thereby reducing asset trading and hence risk sharing.

In this paper, the effect of monetary policy on aggregate welfare is therefore twofold: not only monetary policy lifts the indeterminacy in prices and in equilibrium allocations which characterizes nominal incomplete asset economies, but it also determines the extent to which prices aggregate and convey the information initially dispersed throughout the economy.

We conduct the welfare analysis in the framework of an *example* and by means of *numerical simulations*. We assume that the central bank, acting as a social planner, selects the level of information disclosure at equilibrium that maximizes the value of an expected *social welfare function*. The central bank process of heterogeneous preference aggregation is taken to be random and manifested through actual policymaking and its correspondent social outcomes.
The fact that individuals are heterogeneous implies that by eliminating uncertainty, that is by implementing a policy leading to a fully revealing rational expectation equilibrium, the central bank implicitly makes a certain group of individuals better off. By contrast, when it implements a monetary policy which allows for some or complete uncertainty, that is it implements a policy leading to a fully non revealing or partially revealing rational expectation equilibrium, the central bank leaves insurance opportunities to some other individuals but it possibly let the first type incur in adverse selection effects.

The paper is organized as follows: section 2 presents the model, defines the notion of equilibrium and describes the parametrizations of the space of economies; section 3 focuses on fully revealing rational expectations equilibria; section 4 deals with fully non revealing equilibria and section 5 with partial revelation. Section 6 introduces monetary policy, section 7 presents the algorithm through which monetary policy may influence the revelation of information at equilibrium, section 8 focuses on the welfare analysis and section 9 contains a numerical example. Section 10 concludes.

# 2 The economy

Consider the standard model of a general equilibrium, pure exchange economy under uncertainty, with asymmetric information. Economic activity occurs over two periods, "today", $t = 1$, and in one of $S$ mutually exclusive states of nature, $s \in \mathbf{S} = \{1, \ldots, S\}$, that realizes "tomorrow", $t = 2$.

There is a finite set of households, $h \in \mathbf{H} = \{1, \ldots, H\}$, denoted by the superscript $h$. At time 1, before trade occurs, every household $h$ receives a private signal $\theta^h$ on some specification of the fundamentals of the economy at time 2. The signal takes value in a finite set $\mathbf{\Theta}^h = \{1, \ldots, \Theta^h\}$. No household can directly observe the private signal received by any another household. The Cartesian product of individual sets of signals, $\mathbf{\Theta} = \mathbf{\Theta}^1 \times \ldots \times \mathbf{\Theta}^H$, defines the space of the states of information. Each state of information $\boldsymbol{\theta} \in \mathbf{\Theta}$, represented by a $H$-dimensional collections $\boldsymbol{\theta} = (\theta^1, \ldots, \theta^h, \ldots, \theta^H)$, provides a complete description of the environment in the second period. Hence $\mathbf{\Theta} = \mathbf{S}$, i.e. the space of the joint private signals coincides with the space of the states of nature.

For every household $h$, it will be useful to consider the set $\mathbf{\Theta}^{-h} = \prod_{k \neq h}^{H} \mathbf{\Theta}^k$ defined by the Cartesian product of the sets of signals of all the households other than him. The typical element of such a set is: $\theta^{-h} \equiv (\theta^1 \ldots, \theta^{h-1}, \theta^{h+1}, \ldots, \theta^H)$. A state of nature may then also be denoted as the pair: $s \equiv \left(\theta^{\,h}, \theta^{-h}\right)$.

Individual private information may also be expressed in terms of a partition $\Sigma^{\,h} \equiv \mathbf{\Theta}^h \times \left\{\mathbf{\Theta}^{-h}\right\}$ over the space $\mathbf{S}$ (or, equivalently, over the space of joint signals $\mathbf{\Theta}$). The typical element of the partition $\Sigma^h$ is denoted by $\boldsymbol{\sigma}^h$. Different joint signals, or states of nature, belong to the same element $\boldsymbol{\sigma}^h \in \Sigma^{\,h}$ only if they contain the same realization $\theta^h \in \mathbf{\Theta}^h$.

Given the state of information $\boldsymbol{\theta}$, in the first period and in every state of nature $s = 1, \ldots, S$ in the second period, there is a spot market where finitely many commodities, $\ell \in \mathbf{L} = \{1, \ldots, L\}$, are traded. Commodities are exchanged only on spot markets, there are no markets for contingent commodities. The commodity space is $\mathbf{R}_{++}^{2LS}$.

Each individual consumption space is a closed, nonempty, bounded from below and convex set $\mathbf{X}^h \subset \mathbf{R}_{++}^{2LS}$. Household $h$'s consumption vector at state $s$, is a $2L$-dimensional column vector $\mathbf{x}^h(s) \equiv (\mathbf{x}_1^h(s), \mathbf{x}_2^h(s))$ where $\mathbf{x}_1^h(s) \equiv (\ldots, \mathbf{x}_{1,\ell}^h(s), \ldots)$ and $\mathbf{x}_2^h(s) \equiv (\ldots, \mathbf{x}_{2,\ell}^h(s), \ldots)$ are $L$-dimensional commodity vectors in the first and in the second period. A signal-invariant first period commodity bundle is $\mathbf{x}_1^h(s) \equiv (\ldots, \bar{x}_{1,\ell}^h, \ldots)$.

The initial endowment of household $h$ is denoted by $\boldsymbol{\omega}^h = \left(\boldsymbol{\omega}_1^h, \boldsymbol{\omega}_2^h\right)$, a strictly positive commodity bundle, $\boldsymbol{\omega}^h \in \mathbf{R}_{++}^{2LS}$.

**Assumption 1** *For every household $h \in \mathbf{H}$, first period endowment is signal invariant: $\boldsymbol{\omega}_1^h(\theta^h) = \boldsymbol{\omega}_1^h(\theta'^{\,h}) = \boldsymbol{\omega}_1^h$ for all $\theta^h, \theta'^{\,h} \in \mathbf{\Theta}^h$, $\theta^h \neq \theta'^{\,h}$.*

First period aggregate endowment, the $H$-dimensional vector $\boldsymbol{\omega}_1 = \sum_{h \in \mathbf{H}} \boldsymbol{\omega}_1^h$, is then state-independent.

Intertemporal trade occurs only through the exchange of financial assets in the first period. There are no short-sale or other restrictions on asset trading. Without loss of generality, we assume that there is just one asset $b$ that in the second period, independently of the prevailing state of nature, yields a strictly positive return $(1 + r)$ denominated in abstract units of account. The selling and buying price of the bond is $q$. The financial asset $b$ is an inside asset and, moreover, $\boldsymbol{\omega}_b^h = 0$ for all $h \in \mathbf{H}$, where $\boldsymbol{\omega}_b^h$ is the initial endowment in assets. By trading $b$, households save or borrow, but are not insured neither against the aggregate risk on future resources, which would require that assets on second period aggregate endowment were available, nor against the relative price effects between time-periods and across states as there are no assets with price dependent payoffs. The market structure is also incomplete with respect to information uncertainty as households do not trade any security with signal-contingent payoff before observing their signals. Across realizations of private information $\boldsymbol{\theta} \equiv s$, a portfolio is $\mathbf{b}^h \equiv (\ldots, b^h(s), \ldots)$.

A system of spot commodity prices is a function $p : \mathbf{S} \rightarrow \mathbf{R}_{+}^{2L}$, a $2LS$-dimensional row vector $\mathbf{p} \equiv (\ldots, \mathbf{p}(s), \ldots)$ with $\mathbf{p}(s) \equiv (\mathbf{p}_1(s), \mathbf{p}_2(s))$ where

$\mathbf{p}_1(s) \equiv (\ldots, p_{1,\ell}(s), \ldots)$ and $\mathbf{p}_2(s) \equiv (\ldots, p_{2,\ell}(s), \ldots)$ are first and second period spot commodity prices at state $s$.

It is a well known result that in two-period nominal asset sequential economies with an incomplete financial market, competitive equilibria generically display $(S+1)$ degrees of indeterminacy in prices and allocations. In particular, $(S-1)$ degrees of indeterminacy, affecting the purchasing power of the asset payoffs, modify the asset span and may translate into a real indeterminacy of equilibrium prices and excess demands, while the 2 remaining degrees of indeterminacy are purely nominal. Since nominal indeterminacy does not affect neither consumption nor portfolio choices, we offset the nominal redundancies by normalizing the asset price to one, $q = 1$, and by imposing that first period price of commodity one, in state of aggregate information number one, be equal to one. The domain of commodity prices is then restricted to the set $\mathbf{P} \equiv \left\{\mathbf{p} \in \mathbf{R}^{2LS}_{++} | \, p_{1,1}(1) = 1\right\}$, a smooth manifold without boundary, diffeomorphic to an open subset of $\mathbf{R}^{2LS-1}$.

## 2.1 Individual behavior and equilibrium

Individuals act competitively, do not make any strategic use of their private information and have rational expectations. This is equivalent to say that the households know the equilibrium price map and that they incorporate in their beliefs the information conveyed by the prices they observe on the market. Since we are dealing with an intertemporal economy, the process of expectation refinement takes place in the first period. After observing first period equilibrium prices, exploiting their knowledge of the equilibrium price map, households formulate forecasts on future equilibrium outcomes. Pietra and Siconolfi [18] have specified the conditions that guarantee that in economies with real indeterminacy, second period equilibria be forecastable utilizing the information provided by first period equilibrium prices.

First period prices are a function $p_1 : \mathbf{S} \rightarrow \mathbf{R}^L_+$; they do not need to be measurable with respect to the field on the state space $\mathbf{S}$ generated by individual private information partitions so that two states of the world $s, s' \in \mathbf{S}$ may not be distinguishable by some individual, i.e. $s, s' \in \boldsymbol{\sigma}^h$, but they may be distinguished by the market. Denote by $\Sigma_p$ the coarsest partition that makes first period spot prices $\mathbf{p}_1(s)$ private information measurable. Its typical element is $\boldsymbol{\sigma}_p$. When a household observes the realization $\mathbf{p}_1(s)$, he may refine his initial private information according to the rule:

$$p_1^{-1}(\mathbf{p}_1(s)) = \{\boldsymbol{\sigma}_p \in \Sigma_p \mid \mathbf{p}_1(\boldsymbol{\sigma}_p) = \mathbf{p}_1(s)\},$$

and, having received the signal $\theta^h \in \boldsymbol{\Theta}^h$, he may exclude the occurrence of the states of the world that do not belong to the set:

$$\boldsymbol{\sigma}^h_p = \left(\theta^h \times \left\{\boldsymbol{\Theta}^{-h}\right\}\right) \cap \; p_1^{-1}(\mathbf{p}_1(s)).$$

The information of household $h$ when he decides on the allocation of his resources across time, can then be expressed as a partition $\Sigma^h_p \equiv \left\{\ldots, \boldsymbol{\sigma}^h_p, \ldots\right\}$

of the state space $\mathbf{S}$ defined by

$$\Sigma_p^h \equiv \Sigma^h \vee \Sigma_p,$$

the join of the partition induced by his private information $\Sigma^h$ and the partition $\Sigma_p$ generated by first period equilibrium prices.

**Assumption 2** *First period consumption bundles $\mathbf{x}_1^h(s)$ and asset trading $b^h(s)$ are $\Sigma_p^h-$ measurable.*

The pair $\left(\mathbf{x}_1^h(s), b^h(s)\right)$ is measurable with respect to a partition $\Sigma_p^h$ of the state space $\mathbf{S}$ as long as $\left(\mathbf{x}_1^h(s), b^h(s)\right) = \left(\mathbf{x}_1^h(s'), b^h(s')\right)$ if $(s, s') \in \boldsymbol{\sigma}_p^h$. The measurability requirement captures the restriction that any contract can only be made contingent on the available information. Enforceability is hence guaranteed.

**Assumption 3** *For any partition $\Sigma_p^h$ of the state space $\mathbf{S}$ (or, equivalently, of the set of joint signals $\boldsymbol{\Theta}$) which is at least as fine as the private information partition $\Sigma^h$, the preferences of household h's are described by an additive and time separable von Neumann-Morgenstern utility function $U^h\left(u_1^h\left(\mathbf{x}_1^h\right), u_2^h(\mathbf{x}_2^h); \Sigma_p^h\right) \in C^\infty$.*

1. *$U^h$ is strictly increasing: $D_x U^h(\mathbf{x}^h) \gg 0$ for all $\mathbf{x}^h \in \mathbf{R}_{++}^{2LS}$, or, equivalently, $\partial U^h(\mathbf{x}^h)/\partial x_\ell^h > 0$ for all $\ell \in 2\mathbf{LS}$, hence it has no critical points, preferences are strictly monotonic and at equilibrium prices are strictly positive;*
2. *$U^h$ is strictly concave: $D_x^2 U^h(\mathbf{x}^h)$ is negative definite for all $x^h \in \mathbf{R}_{++}^{2SL}$, that is: $[D_x^2 U^h(\mathbf{x}^h)]h^T < 0$ for every $h \neq 0$, $h \in \mathbf{R}^{2SL}$, when restricted to the tangent hyperplane to the indifference surface through every $\mathbf{x}^h \in \mathbf{R}_{++}^{2LS}$ : $\left\{h \in \mathbf{R}^{2SL} | D_x U^h(\mathbf{x}^h)h = 0\right\}$. Therefore, the indifference surface through every $\mathbf{x}^h \in \mathbf{R}_{++}^{2LS}$ has a nowhere zero Gaussian curvature, which implies that individuals are risk averse and that among the commodities there are no perfect substitutes;*
3. *$U^h$ satisfies the boundary conditions: for any real number $c \in \mathbf{R}$, the indifference surface $U^{-1}(c)$ is closed in $\mathbf{R}_{++}^{2LS}$, $\left\{(\mathbf{x}^h) \in \mathbf{R}_{++}^{2LS} | U^h(\mathbf{x}^h) \geqq U^h(\hat{\mathbf{x}}^h)\right\}$ for all $(\hat{\mathbf{x}}^h) \in \mathbf{R}_{++}^{2LS}$, so that any indifference surface passing through a strictly positive consumption bundle does not intersect the boundary of non-negative orthants.*

Assumptions 3.1-3.3 are standard and yield well-defined, point-valued and differentiable excess demand functions.

At private signal $\theta^h \in \boldsymbol{\Theta}^h$, after observing first period spot prices $\mathbf{p}_1(s)$, every household $h \in \mathbf{H}$ chooses a consumption plan $\mathbf{x}^h : \boldsymbol{\sigma}_p^h \to \mathbf{R}_{++}^{L+L\boldsymbol{\sigma}_p^h}$ and an asset trading $b^h : \boldsymbol{\sigma}_p^h \to \mathbf{R}$ as a solution of the following *ex-post* programming problem:

$$\max_{x^h, b^h} \quad U^h(\mathbf{x}^h; \Sigma_p^h) = \sum_{s \in \boldsymbol{\sigma}_p^h} \pi(s|\boldsymbol{\sigma}_p^h) \left[u_1^h\left(\mathbf{x}_1^h(\boldsymbol{\sigma}_p^h)\right) + u_2^h\left(\mathbf{x}_2^h(s)\right)\right],$$

subject to the budget constraints:

$$
\begin{aligned}
\mathbf{p}_1(s) \cdot \mathbf{z}_1^h(s) + b^h(s) = 0, \quad s \in \boldsymbol{\sigma}_p^h, \\
\mathbf{p}_2(s) \cdot \mathbf{z}_2^h(s) - (1+r)b^h(s) = 0, \quad s \in \boldsymbol{\sigma}_p^h, \\
(\mathbf{z}_1^h, b^h) \ s\text{-invariant in } \boldsymbol{\sigma}_p^h,
\end{aligned} \tag{1}
$$

where $\mathbf{z}_t^h(s) \equiv (\mathbf{x}_t^h(s) - \boldsymbol{\omega}_t^h(s))$, $t = 1, 2$ are the individual excess demand vectors.

It will be more convenient to formulate maximization problems *ex-ante:* before any household $h$ observes his private signal $\theta^h$ and the realization $\mathbf{p}_1(s) \in \mathbf{P}$, individual consumption plan $\mathbf{x}^h : \mathbf{S} \to \mathbf{R}_{++}^{L+LS}$ and asset trading $b^h : \mathbf{S} \to \mathbf{R}$ choices are defined by the solution of the following optimization problem:

$$
\max_{x^h, b^h} \quad U^h(\mathbf{x}^h; \Sigma_p^h) = \sum_{\boldsymbol{\sigma}_p^h \in \Sigma_p^h} \pi(\boldsymbol{\sigma}_p^h) \sum_{s \in \boldsymbol{\sigma}_p^h} \pi^h(s|\boldsymbol{\sigma}_p^h) \left[ u_1^h(\mathbf{x}_1^h(\boldsymbol{\sigma}_p^h)) + u_2^h(\mathbf{x}_2^h(s)) \right],
$$

subject to the budget constraints:

$$
\begin{aligned}
\mathbf{p}_1(s) \cdot \mathbf{z}_1^h(s) + b^h(s) &= 0, \quad s \in \boldsymbol{\sigma}_p^h, \ \ \boldsymbol{\sigma}_p^h \in \Sigma_p^h, \\
\mathbf{p}_2(s) \cdot \mathbf{z}_2^h(s) - (1+r)b^h(s) &= 0, \quad s \in \mathbf{S}, \\
(\mathbf{z}_1^h, b^h) \ \ \Sigma_p^h &- \text{measurable.}
\end{aligned} \tag{2}
$$

Since expected utility function $U^h$ is intertemporally additive and linear in probabilities and because of the structure of the budget constraints, the optimal solution to (2) is just the collection of the optimal solutions to (1). Hence, every individual ex-ante programming problem (2) decomposes into as many ex post programming problems (1) as are the elements $\boldsymbol{\sigma}_p^h$ of his information partition $\Sigma_p^h$. Evidently, the finer the partition, the smaller the uncertainty a household has to face, i.e. the fewer the states of nature $s \in \boldsymbol{\sigma}_p^h$ he must take into account when solving for his optimization problem.

**Definition 4** ***Rational Expectations Equilibrium (ree)*** *A rational expectations equilibrium (ree), is a system of spot prices $\mathbf{p}^* \in \mathbf{P}$ with associated commodity and asset allocations $(\mathbf{x}^*, b^*) \equiv (\ldots, (\mathbf{x}^{h*}, b^{h*}), \ldots)$ such that: i) $(\mathbf{x}^{h*}, b^{h*}) \equiv (\ldots, \mathbf{x}^{h*}(s), b^{h*}(s), \ldots)$ solves each individual ex-ante programming problem (2); at $\mathbf{p}^*$, in each state of nature $s \in \mathbf{S}$, ii) first and second period commodity markets clear, $\sum_{h \in \mathbf{H}} \mathbf{x}_1^{h^*}(s) = \sum_{h \in \mathbf{H}} \boldsymbol{\omega}_1^h$ and $\sum_{h \in \mathbf{H}} \mathbf{x}_2^{h^*}(s) = \sum_{h \in \mathbf{H}} \boldsymbol{\omega}_2^h(s)$, iii) the asset market clears, $\sum_{h \in \mathbf{H}} b^{h^*}(s) = 0$.*

## 2.2 The space of economies

In the sequel we will parametrize the space of economies both, in terms of utility functions, and in terms of endowments.

### 2.2.1 Parametrization in terms of utilities

Denote by $\mathbf{U}^*$ the topological space of the Bernoulli utility functions $u_1^h, u_2^h$ satisfying assumptions 3.1 - 3.3. The space $\mathbf{U}^*$ is endowed with the $C^2$ compact - open (weak) topology (see Mas-Colell [13]). Call $\mathbf{U} \equiv (\mathbf{U}^*)^H$ the topological space endowed with the product topology. An economy $u$ is a collection $u \equiv \{u_1^h, u_2^h\}_{h=1}^H$ of Bernoulli utility functions that belongs to the topological space $\mathbf{U}$. The space $\mathbf{U}$ is a metrizable, separable and complete topological space. Since we will use small, (locally) finite dimensional, linear perturbations of the utility functions, when we perturb (or differentiate) Bernoulli utilities, we take $\mathbf{U}$ to be a finite dimensional manifold (see Mas-Colell, p. 301 [13], Pietra [17] and Pietra and Siconolfi [18]).

The representation of preferences by means of an intertemporally separable von Neumann-Morgenstern utility function, implies that first period Bernoulli utility functions $u_1^h$ are state invariant, i.e. that they are invariant across permutations of first period consumption plans related to different realizations of the joint signal. We will then distinguish between first and second period utilities. Consider household $h$'s equilibrium consumption plans $\mathbf{x}^{h*}(s), \mathbf{x}^{h*}(s')$ in any two states $s, s' \in \mathbf{S} : \mathbf{x}^{h*}(s) \equiv \left(\mathbf{x}_1^{h*}(s), \mathbf{x}_2^{h*}(s)\right)$ and $\mathbf{x}^{h*}(s') \equiv \left(\mathbf{x}_1^{h*}(s'), \mathbf{x}_2^{h*}(s')\right)$.

For *first period* utility function $u_1^h$ to be perturbable, $h$'s equilibrium consumption plans in states $s, s'$ must differ: $\mathbf{x}_1^{h*}(s) \neq \mathbf{x}_1^{h*}(s')$. Recall that first period consumption plans are strictly positive vectors of dimension $L$. Consider two open neighborhoods $B_{1,j}^h$, $j = s, s'$ of $\mathbf{x}_1^{h*}(s)$ and $\mathbf{x}_1^{h*}(s')$ with closures included in the interior of the $L$-dimensional vector space: $\mathrm{Cl}\, B_{1,j}^h \subset \mathbf{R}_{++}^L$. Select two positive scalars, $\varepsilon_j$, as radiuses of the open balls $B_{1,\varepsilon_s}^h\left(\mathbf{x}_1^{h*}(s)\right)$ and $B_{1,\varepsilon_{s'}}^h\left(\mathbf{x}_1^{h*}(s')\right)$ in a way that satisfies both, the property that $\mathrm{Cl}\, B_{1,j}^h \subset B_{1,\varepsilon_j}^h \subset \mathrm{Cl}\, B_{1,\varepsilon_j}^h \subset \mathbf{R}_{++}^L$, and the property that $\mathrm{Cl}\, B_{1,\varepsilon_s}^h \cap \mathrm{Cl}\, B_{1,\varepsilon_{s'}}^h = \emptyset$, i.e. that the closures of the open balls $B_{1,\varepsilon_s}^h\left(\mathbf{x}_1^{h*}(s)\right)$ and $B_{1,\varepsilon_{s'}}^h\left(\mathbf{x}_1^{h*}(s')\right)$ be disjoint.

Similarly, in order to perturb *second period* utility function $u_2^h$, consider two open neighborhoods $B_{2,\epsilon_s}^h\left(\mathbf{x}_2^{h*}(s)\right)$ and $B_{2,\epsilon_{s'}}^h\left(\mathbf{x}_2^{h*}(s')\right)$ of equilibrium allocations $\mathbf{x}_2^{h*}(s)$ and $\mathbf{x}_2^{h*}(s')$, such that $\mathrm{Cl}\, B_{2,j}^h \subset B_{2,\epsilon_j}^h \subset \mathrm{Cl}\, B_{2,\epsilon_j}^h \subset \mathbf{R}_{++}^L$, $j = s, s'$ and $\mathrm{Cl}\, B_{2,\epsilon_s}^h \cap \mathrm{Cl}\, B_{2,\epsilon_{s'}}^h = \emptyset$.

Given a collection of individual utilities $u \in \mathbf{U}$, the open sets $B_{1,j}^h$, $B_{2,j}^h$ and $B_{1,\varepsilon_j}^h\left(\mathbf{x}_1^{h*}(j)\right)$, $B_{2,\varepsilon j}^h\left(\mathbf{x}_2^{h*}(j)\right)$ and four strictly positive vectors of parameters $\boldsymbol{\delta}_{t,j}^h \in \mathbf{R}_{++}^L$, $t = 1, 2$, $j = s, s'$, call $u_\delta$ the economy where individual Bernoulli utilities $u_{1,\delta}^h$ are $u_{2,\delta}^h$ are obtained by replacing utilities $u_1^h$ and $u_2^h$ with the functions,

$$\begin{aligned}
u_{1,\delta}^h\left(\mathbf{x}_1^h(j)\right) &\equiv u_1^h\left(\mathbf{x}_1^h(j)\right) + \phi_{1,j}\left(\mathbf{x}_1^h, B_{1,\varepsilon j}^h\left(\mathbf{x}_1^{h*}(j)\right)\right)(\boldsymbol{\delta}_{1,j}\,\mathbf{x}_1^h), \qquad (3)\\
u_{2,\delta}^h\left(\mathbf{x}_2^h(j)\right) &\equiv u_2^h\left(\mathbf{x}_2^h(j)\right) + \phi_{2,j}\left(\mathbf{x}_2^h, B_{2,\epsilon_j}^h\left(\mathbf{x}_2^{h*}(j)\right)\right)(\boldsymbol{\delta}_{2,j}^h\,\mathbf{x}_2^h),
\end{aligned}$$

where $\phi_{1,j}(\cdot)$ and $\phi_{2,j}(\cdot)$ are smooth "bump" functions $\phi : \mathbf{R}_{++}^L \to [1,2]$ that take value 1 if $\mathbf{x}_1^h \in B_{1,j}^h$ (and $\mathbf{x}_2^h \in B_{2,j}^h$) and value 0 if $\mathbf{x}_1^h \notin \mathrm{Cl}\, B_{\varepsilon_j}^h$ (or if $\mathbf{x}_2^h \notin \mathrm{Cl}\, B_{\epsilon_j}^h$), $j = s, s'$, (see Hirsch, p. 41 [11]). Since the perturbations on $u_1^h, u_2^h$ in $B_{1,j}^h$, $B_{2,j}^h$ are locally finite and linear, they smoothly vanish outside $B_{1,\varepsilon_j}^h$ and $B_{2,\epsilon_j}^h$, $j = s, s'$. Moreover, since $B_{1,\varepsilon_j}^h$, $B_{2,\epsilon_j}^h$ are open neighborhoods of $\mathbf{x}_1^{h*}(s)$, $\mathbf{x}_2^{h*}(s)$ larger than $B_{1,j}^h$, $B_{2,j}^h$ the utility function perturbations leave the boundary conditions satisfied. The Hessian matrices are not affected as both, $D_{\mathbf{x}}\,\phi_{1,j}[\mathbf{x}_1^h, B_{1,\varepsilon_j}^h] = 0$ and $D_{\mathbf{x}}^2\,\phi_{1,j}[\mathbf{x}_1^h, B_{1,\varepsilon_j}^h] = 0$ at $\mathbf{x}_1^{h*}(j)$; also $D_{\mathbf{x}}\,\phi_{2,j}[\mathbf{x}_2^h, B_{2,\epsilon_j}^h] = 0$ and $D_{\mathbf{x}}^2\,\phi_{2,j}[\mathbf{x}_2^h, B_{2,\epsilon_j}^h] = 0$ at $\mathbf{x}_2^{h*}(j)$, $j = s, s'$.

When functions $u_1^h$, $u_2^h$ satisfy the conditions 3.1 -3.3, so will do functions $u_{1,\delta}^h$, $u_{2,\delta}^h$. If parameters $\boldsymbol{\delta}_{t,j}^h \in \mathbf{R}_{++}^L$, $t = 1, 2$, $j = s, s'$ are appropriately chosen and are small enough, the economy $u_\delta$ is arbitrarily close to the original economy $u$. Hence also the modified utility functions belong to the space $\mathbf{U}^*$, i.e. $u_\delta = \left\{u_{1,\delta}^h, u_{2,\delta}^h\right\} \in \mathbf{U}^*$. In the sequel, to establish that a property holds for a dense set of economies, it will then suffice to show that it holds for arbitrary small, locally finite and linear perturbations of the Bernoulli utility functions.

### 2.2.2 Parametrization in terms of endowments

We will parametrize the space of economies also in terms of endowments. In this case, the space of economies endowed with the product topology is denoted $\boldsymbol{\Omega}$ and is defined as

$$\boldsymbol{\Omega} \equiv \left\{\boldsymbol{\omega} \in \mathbf{R}_{++}^{L(1+S)H} \mid \boldsymbol{\omega}_1^h(\theta^h) \ \ \theta^h\textit{-invariant for all } h \in \mathbf{H}\right\}.$$

# 3 Fully revealing ree

**Definition 5** ***Fully revealing ree*** *A rational expectations equilibrium of an economy $u \in \mathbf{U}'$, $\mathbf{U}' \subset \mathbf{U}$, is fully revealing if first period equilibrium prices differ across states:*

$$\mathbf{p}_1(s) \neq \mathbf{p}_1(s') \qquad \textit{for all } \ s, s' \in \mathbf{S} \qquad \textit{when } s \neq s'.$$

Remark

When nominal assets are more than one and the financial market is incomplete, the existence of fully revealing rational expectations equilibria as defined by the classical one-to-one property with aggregate information, is always guaranteed. The indeterminacy of equilibrium asset prices allows in fact to choose any nonarbitrage asset price in a way that equilibrium prices be always injective in joint signals. Polemarchakis and Siconolfi [21] introduced then the more demanding notion of strongly fully revealing equilibria that requires that at least one equilibrium price be associated with just a single joint signal. They also showed, however, that because of the indeterminacy of equilibrium prices that characterizes nominal asset economies, strongly fully revealing equilibria do not exist. To characterize the sets of equilibrium allocations for different degrees of revelation, a single nominal asset suffices. Its price normalization yields an economy with real assets in which, under the assumption of a finite number of states, any fully revealing equilibria is also a strongly revealing one. The generic existence of fully revealing equilibria in non trivial sequential economies with real assets has been proved in the endowment space by Pietra and Siconolfi [19] and by Stahn [24]. The argument of existence is an essential step to characterize the dimension of the set of fully revealing equilibria. Because of the assumption of signal invariance of first period endowment that constrains the endowment set to a vector space of lower dimension than the consumption set, and the assumption of first period signal invariance of Bernoulli utility function, we will prove the existence of fully revealing equilibria with a transversality argument based on preference perturbations.

## 3.1 Existence of fully revealing ree

First period fully revealing equilibrium prices, pooling the joint signal, decompose the economy into as many as $S$ distinct subeconomies. The rational expectation hypothesis requires, in fact, that the households extract from fully revealing market prices all the information they need to disclose the state of the world to realize.

**Lemma 6** *In any pair of full-communication subeconomies* $s,s' \in \mathbf{S}$*,* $s \neq s'$*, there exists at least one individual* $h = 1$ *with* $s$ *- dependent first period excess demand functions at equilibrium:* $\mathbf{z}_1^{1*}(s) \neq \mathbf{z}_1^{1*}(s')$.

**Proof :** Consider a full information economy and suppose that there exist any two states $s, s' \in \mathbf{S}$ , $s \neq s'$ in which at equilibrium: $\mathbf{z}_1\left(\mathbf{p}_1(s); \boldsymbol{\omega}_1, \Sigma_p^h\right) - \mathbf{z}_1\left(\mathbf{p}_1(s'); \boldsymbol{\omega}_1, \Sigma_p^h\right) = \mathbf{0}$ and $\mathbf{p}_1(s) - \mathbf{p}_1(s') = \mathbf{0}$, where $\mathbf{z}_1$ is the aggregate excess demand function for first period commodities. Since $s \neq s'$, the conditions above are satisfied if every household's information structure distinguishes between $s$ and $s'$ so that nobody needs to learn anything from prices. This hypothesis is excluded by assumption. If there exists at least one household, say $h = 1$, without such a fine information partition, the conditions are satisfied if $h = 1$ extracts no information from first period prices. Given the assumption on rational expectations, this would be possible only if the economy did not assign to the distinction between states $s, s'$ any relevance. This possibility is excluded by construction: only the events which are distinguished by the pooled, aggregate information and that influence households' preferences and consumption plans are consistent with the definition of fully revealing rational expectation equilibrium. □

**Proposition 7** *There is an open and dense set of economies* $\mathbf{U}' \subset \mathbf{U}$*, such that for each* $u \in \mathbf{U}'$ *every subeconomy* $s$ *is regular. Hence fully revealing rational expectations equilibria exist.*

**Proof :** At fully revealing prices households disclose the state to occur already in the first period. This implies that, when solving for their *ex-post* optimization problem, they face a *unique* budget constraint as if the asset market were complete (see Cass [4]). The commodity prices in every subeconomy are then restrained to the set $\mathbf{P}' = \left\{p \in \mathbf{R}^{2L}_{++} | p_{1,1} = 1\right\}$, a smooth manifold without boundaries diffeomorphic to an open subset of $\mathbf{R}^{2L-1}$. By definition, the subeconomy $s$

is regular if and only if $\mathbf{0} \in \mathbf{R}^{(2L-1)}$ is a regular value of the aggregate excess demand function $\mathbf{z}_s\left(\mathbf{p}(s);\boldsymbol{\omega}(s),\Sigma_p^h\right) \equiv \left(\mathbf{z}_1\left(\mathbf{p}_1(s);\boldsymbol{\omega}_1,\Sigma_p^h\right),\mathbf{z}_2\left(\mathbf{p}_2(s);\boldsymbol{\omega}_2(s),\Sigma_p^h\right)\right)$. Note that the price set $\mathbf{P}'$ is compact because the aggregate consumption set is closed, preferences are strictly monotonic, the aggregate excess demand function $\mathbf{z}(\cdot)$ is continuous and satisfies the boundary conditions.

If subeconomies $s$ and $s'$, $s, s' \in \mathbf{S}$, $s \neq s'$ are *regular*, the equation system

$$\begin{aligned}
\mathbf{z}_s\left(\mathbf{p}_1^*(s),\mathbf{p}_2^*(s);\boldsymbol{\omega}_1,\boldsymbol{\omega}_2(s),\Sigma_p^h\right) &= \mathbf{0}, \quad 2L-1 \text{ equations}, \\
\mathbf{z}_{s'}(\mathbf{p}_1^*(s'),\mathbf{p}_2^*(s');\boldsymbol{\omega}_1,\boldsymbol{\omega}_2(s'),\Sigma_p^h) &= \mathbf{0}, \quad 2L-1 \text{ equations}, \\
\mathbf{p}_1^*(s)-\mathbf{p}_1^*(s') &= \mathbf{0}, \quad L \text{ equations},
\end{aligned} \tag{4}$$

admits no solution because it has $2(2L-1)$ independent commodity prices with $(5L-2)$ linearly independent equations. We show that, generically, system *(4)* has no solution, and hence subeconomies $s$ and $s'$ are regular, by means of a transversality argument based on the perturbation of a household's Bernoulli utility functions. Denote by $z$ the function: $z: \mathbf{P}' \times \mathbf{P}' \times \mathbf{U} \to \mathbf{R}_{++}^{5L-2}$ with $z(\cdot) \equiv (\mathbf{z}_s(\cdot), \mathbf{z}_{s'}(\cdot), (\mathbf{p}_1^*(s)-\mathbf{p}_1^*(s'))) = \mathbf{0}$. If at fully revealing equilibrium prices $\mathbf{p}^* \in \mathbf{P}'$ subeconomies $s$ and $s'$ are regular, the function $z$ is transverse to zero, $z \pitchfork 0$, i.e. the matrix of price effects $Dz|_p$ is nonsingular and has maximal rank $2(2L-1)$.

By lemma *6* there exists at least one individual $h=1$ whose first period excess demand functions at equilibrium are $s$ -dependent: $\mathbf{z}_1^{1*}(s) \neq \mathbf{z}_1^{1*}(s')$. Since first period endowments are state-invariant by assumption, $h=1's$ first period consumption plans must be $s$ -dependent: $\mathbf{x}_1^{1*}(s) \neq \mathbf{x}_1^{1*}(s')$. As already explained in section 2.2.1, we may set some parameters $\boldsymbol{\delta}_{t,j}^1 \in \mathbf{R}_{++}^L$, $t=0,1$, $j=s,s'$, choose appropriate scalars $\varepsilon_j, \epsilon_j \in \mathbf{R}$, $j=s,s'$ and define "bump" functions $\phi_{t,j}[\cdot]$, $t=0,1$, $j=s,s'$ and replace $h=1$'s *ex-post* utility functions $u^1(\mathbf{x}^1(s))$ and $u^1(\mathbf{x}^1(s'))$ with functions $u_\delta^1(\mathbf{x}^1(s)) \equiv u_{1,\delta}^1(\mathbf{x}_1^1(s)) + u_{2,\delta}^1(\mathbf{x}_2^1(s))$ and $u_\delta^1(\mathbf{x}^1(s')) \equiv u_{1,\delta}^1(\mathbf{x}_1^1(s')) + u_{2,\delta}^1(\mathbf{x}_2^1(s'))$, where $u_{1,\delta}^1$, $u_{2,\delta}^1$ are defined in (3).

By assumption, the set of solutions $\mathbf{x}^{1*}(j), \mathbf{p}_1^*(j), \mathbf{p}_2^*(j)$, $j=s,s'$ solves the following necessary and sufficient conditions of $h=1$'s *ex-post* optimization problem:

$$\begin{aligned}
D_{\mathbf{x}_1}u_1^1(\mathbf{x}_1^{1\,*}(j)) + \boldsymbol{\delta}_{1,j}^1 - \lambda^1(j)\mathbf{p}_1^*(j) &= \mathbf{0}, \quad L \text{ equations}, \\
D_{\mathbf{x}_2}u_2^1(\mathbf{x}_2^{1*}(j)) + \boldsymbol{\delta}_{2,j}^1 - \lambda^1(j)\mathbf{p}_2^*(j) &= \mathbf{0}, \quad L \text{ equations}, \\
\mathbf{p}_1^*(j)\cdot\mathbf{z}_1^{1*}(j) + \mathbf{p}_2^*(j)\cdot\mathbf{z}_2^{1*}(j) &= 0, \quad 1 \text{ equation}.
\end{aligned}$$

By small arbitrary perturbations of the vectors $\boldsymbol{\delta}_{t,j}^1 \in \mathbf{R}_{++}^L$ $j=s,s'$, $t=1,2$, provided that for each $\mathbf{x}_1^{1*}(j) \in B_{1,\varepsilon_j}^1$ and for each $\mathbf{x}_2^{1*}(j) \in B_{2,\epsilon_j}^1$ the "bump" functions take values $\phi_{1,j}[\mathbf{x}_1^1, B_{1,\varepsilon_j}^1] = 1$ and $\phi_{2,j}[\mathbf{x}_2^1, B_{2,\epsilon_j}^1] = 1$, we make the determinant of the Jacobian $D_{\boldsymbol{\delta}_j^1} z_t^1$, different from zero:

$$\left|D_{\boldsymbol{\delta}_j^1}\, z_t^1\right|_{j=s,s'\;t=1,2} = \left|\begin{matrix} \frac{\partial z_t^1}{\partial \delta^1_{1,j,1}} & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & \frac{\partial z_t^1}{\partial \delta^1_{1,j,L}} & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & \frac{\partial z_t^1}{\partial \delta^1_{2,j,\ell}} & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & \frac{\partial z_t^1}{\partial \delta^1_{2,j,L-1}} \end{matrix}\right| \neq 0.$$

Hence matrices $D_{\boldsymbol{\delta}^1_{i,j}}\, z^1_j(\cdot)$, $j = s, s'$, $t = 1, 2$ are nonsingular. As $\mathbf{z}_j(\cdot) \equiv \sum_{h\in\mathbf{H}} z^h_j(\cdot)$ implies that $D_{\delta^h_{i,j}}\mathbf{z}_j(\cdot) = \sum_{h\in\mathbf{H}} D_{\delta^h_{i,j}} z^h_j(\cdot) = D_{\delta^1_{i,j}} z^1_j(\cdot)$, if $D_{\delta^1_{i,j}} z^1_j(\cdot)$ is non singular, so is the Jacobian matrix of the aggregate demand function, $D_{\delta^h_{i,j}}\mathbf{z}_j(\cdot)$.

Denote by $\mathbf{A}$ the $(5L-2)\times(8L-4)$ dimensional Jacobian of the map $z$ estimated at the equilibrium solutions $\mathbf{x}^{1*}_t(j)$, $\mathbf{p}^*_t(j)$, $t = 1, 2$, $j = s, s'$ :

$$A \equiv \begin{bmatrix} D_{\mathbf{p}} z_s & 0 & D_{\boldsymbol{\delta}^{1*}_t} z_s & 0 \\ 0 & D_{\mathbf{p}} z_{s'} & 0 & D_{\boldsymbol{\delta}^{1*}_t} z_{s'} \\ \mathbf{I}_L & -\mathbf{I}_L & 0 & 0 \end{bmatrix}$$

where $D_{\mathbf{p}} z_j = \begin{bmatrix} D_{\mathbf{p}^*_1} z_{1,j} & D_{\mathbf{p}^*_2} z_{1,j} \\ D_{\mathbf{p}^*_1} z_{2,j} & D_{\mathbf{p}^*_2} z_{2,j} \end{bmatrix}$ and $D_{\boldsymbol{\delta}^{1*}_{t,j}} Z_j = \begin{bmatrix} D_{\boldsymbol{\delta}^{1*}_1} z_{1,s} & 0 \\ 0 & D_{\boldsymbol{\delta}^{1*}_2} z_{2,s} \end{bmatrix}$,

$t = 1, 2$, $j = s, s'$. Since matrices $D_{\boldsymbol{\delta}^{1*}_{t,j}} Z_j$ have maximal rank by construction and the matrices $\mathbf{I}$ are the $L$-dimensional identity matrices, the $(5L-2)$ rows of the matrix $\mathbf{A}$ are linearly independent. This implies that the function $z$ is transversal to zero: $z \pitchfork 0$.

Denote $z_\delta$, the map $z$ for a given economy $u \in \mathbf{U}$. By the joint transversality theorem, the function $z_\delta : \mathbf{P}' \times \mathbf{P}' \to \mathbf{R}^{5L-2}_{++}$ is transversal to zero, $z_\delta \pitchfork 0$, for a dense and of full Lebesgue measure set, say the set $\boldsymbol{\Delta}' \subset \boldsymbol{\Delta}$, of parameter vectors $\boldsymbol{\delta}^1_{t,j}$, $j = s, s'$, $t = 1, 2$, influencing individual $h = 1$'s preferences. Since $z_\delta^{-1}(0) \cap \mathbf{P}'$ is a compact set, by the transversality theorem, $z_\delta \pitchfork 0$ implies that there is an open and dense subset of economies $\mathbf{U}' \subset \mathbf{U}$, such that $z_\delta^{-1}(0) = \emptyset$. Given that the domain of the map $z_\delta$ is a vector space of dimension $2(2L-1)$, while its range is a vector space of dimension $(5L-2)$, $z_\delta$ is transversal to zero if and only if $z_\delta^{-1}(0) \cap \mathbf{P}'$ is empty. This implies that, system (4) has no solution.

Since the perturbations of $h = 1$'s preferences typically guarantee that the Jacobians with respect to equilibrium prices of the aggregate excess demand functions $D_{\mathbf{p}}\mathbf{z}_j\left(\mathbf{p}^*_1(j), \mathbf{p}^*_2(j); \boldsymbol{\omega}_1, \boldsymbol{\omega}_2(j), \Sigma^h_p\right)$, $j = s, s'$ have maximal rank, by the implicit function theorem, first period equilibrium spot prices are locally unique. Hence fully revealing rational expectations equilibria exist. $\square$

## 3.2 The set of fully revealing ree

**Proposition 8** *For every normalized economy $u \in \mathbf{U}'$, the set of fully revealing rational expectations equilibria contains a zero-dimensional submanifold and consists of $S$ of locally unique competitive equilibria.*

**Proof :** Given the structure of households' optimization problems, at fully revealing first period prices, the economy decomposes into as many disjoint subeconomies as there are distinct joint private signals, or states of nature, i.e. $S$. The result follows then from Proposition 7 and the regularity of each subeconomy. $\square$

# 4 Fully non revealing ree

A rational expectations equilibrium is fully noninformative if there is no informational feedback from first period equilibrium prices.

**Definition 9** ***Fully non revealing ree*** *A rational expectations equilibrium for an economy $\omega \in \mathbf{\Omega}$ is fully noninformative if first period commodity prices are state invariant:*

$$\mathbf{p}_1(s) = \mathbf{p}_1(s') = \mathbf{p}_1 \qquad \textit{for all} \;\; s, s' \in \mathbf{S} \qquad \textit{when } s \neq s'. \tag{5}$$

The existence of fully non revealing rational expectations equilibria has been proved by Polemarchakis and Siconolfi [21].

## 4.1 The set of fully non revealing ree

**Proposition 10** *There is a (relatively) open, dense set of full Lebesgue measure $\mathbf{\Omega}' \subset \mathbf{\Omega}$, such that for every economy $\omega \in \mathbf{\Omega}'$ the set of fully non revealing rational expectations equilibria contains a submanifold of dimension*

$$(S-1) - \sum_{h \in \mathbf{H}} \left(\Theta^h - 1\right).$$

**Proof:** We break the argument into two steps.

**Step 1:** We establish that fully noninformative economies are typically regular.
At fully non revealing prices the information available to household $h$ coincides with his own private information, $\sigma^h \in \Sigma^h \equiv \mathbf{\Theta}^h \times \left\{\mathbf{\Theta}^{-h}\right\}$. The individual ex-ante programming problem is then:

$$\max_{x^h, b^h} \quad U^h(\mathbf{x}^h; \mathbf{\Theta}^h) = \sum_{\theta^h \in \mathbf{\Theta}^h} \pi^h(\theta^h) \sum_{s \in \boldsymbol{\sigma}^h} \pi(s|\theta^h) \left[u_1^h(\mathbf{x}_1^h(\theta^h)) + u_2^h(\mathbf{x}_2^h(s))\right],$$

subject to:

$$\begin{aligned}
\mathbf{p}_1 \cdot \mathbf{z}_1^h\left(\theta^h, \theta^{-h}\right) + b^h\left(\theta^h, \theta^{-h}\right) &= 0, \quad \left(\theta^h, \theta^{-h}\right) \equiv s, \quad \theta^h \in \mathbf{\Theta}^h, \quad (6)\\
\mathbf{p}_2(s) \cdot \mathbf{z}_2^h(s) - (1+r)b^h(s) &= 0, \quad s \in \mathbf{S},\\
(\mathbf{z}_1^h, b^h) \;\; \Sigma^h &- \textit{measurable}.
\end{aligned}$$

As in Mischel, Polemarchakis, Siconolfi [16] and Polemarchakis and Siconolfi [21], we first consider an economy where budget constraints have been modified. Consider the following *ex-ante* individual optimization problem with a "modified" budget constraint:

$$\max_{x^h, b^h} \quad U^h(\mathbf{x}^h; \mathbf{\Theta}^h) = \sum_{\theta^h \in \mathbf{\Theta}^h} \pi^h(\theta^h) \sum_{s \in \boldsymbol{\sigma}^h} \pi(s|\theta^h)\left[u_1^h(\mathbf{x}_1^h(\theta^h)) + u_2^h(\mathbf{x}_2^h(s))\right],$$

subject to:

$$\begin{aligned}
\sum_{\theta^h \in \mathbf{\Theta}^h} \pi^h(\theta^h)\left[\mathbf{p}_1 \cdot \mathbf{z}_1^h\left(\theta^h, \theta^{-h}\right) + b^h\left(\theta^h, \theta^{-h}\right)\right] &= 0, \qquad (7)\\
\mathbf{p}_2(s) \cdot \mathbf{z}_2^h(s) - (1+r)b^h(\theta^h) &= 0, \quad s \in \mathbf{S},\\
(\mathbf{z}_1^h, b^h) \;\; \Sigma^h - \textit{measurable},
\end{aligned}$$

where first period budget constraint is the sum of as many as $\Theta^h$ fist period budget constraints of planning problem (6). Call *modified economy* the economy in which individuals solve programming problem (7). Observe that if household $h$ does not learn anything from prices, the measurability constraint in the maximization problem makes his excess demands for first period commodities and assets $\theta^{-h}$ - invariant: $z_1^h\left(\theta^h, \theta^{-h}\right) = z_1^h\left(\theta^h\right)$ and $b^h\left(\theta^h, \theta^{-h}\right) = b^h\left(\theta^h\right)$.

**Lemma 11** *At fully noninformative first period commodity prices, first period individual excess demand functions $z_1^h\left(\theta^h\right)$ are signal-invariant: $\mathbf{z}_1^h(\theta^h) = \mathbf{z}_1^h$ for all $\theta^h \in \mathbf{\Theta}^h$, all $h \in \mathbf{H}$.*

**Proof:** Given the intertemporal separability and strict concavity of utility functions together with the signal-independence of first period utility functions and endowments, the signal invariance of individual excess demand functions $\mathbf{z}_1^h(\theta^h)$ follows from Jensen inequality. □

As a consequence of Lemma 11, in the modified economy, first period budget constraint in any individual ex-ante optimization problem becomes

$$\mathbf{p}_1 \cdot \mathbf{z}_1^h + \sum_{\theta^h \in \mathbf{\Theta}^h} \pi^h(\theta^h) b^h(\theta^h) = 0. \qquad (8)$$

Note that at fully non revealing equilibrium prices, individual bond functions are also signal invariant, $b^h(\theta^h) = b^h(\theta'^h)$, because any excess demand for bonds must satisfy the budget constraints given prices and because of the $\Sigma^h$-measurability constraint on $(\mathbf{z}_1^h, b^h)$. As a consequence, the first period budget constraint in the *ex-post* programming problem becomes: $\mathbf{p}_0 \cdot \mathbf{z}_0^h + b^h = 0$. But then, individual optimal solutions at fully non revealing prices belong to the budget sets of the original economy since

$$\mathbf{p}_1 \cdot \mathbf{z}_1^h(\theta^h) + b^h(\theta^h) = \mathbf{p}_1 \cdot \mathbf{z}_1^h + b^h = 0, \qquad \textit{for all} \ \ \theta^h \in \mathbf{\Theta}^h.$$

However, the budget set of the modified economy contains the budget set of the original economy, which means that any set of noninformative rational expectations prices that are equilibrium prices for the modified economy are also equilibrium prices for the original economy. Hence we can focus on the modified economy as the competitive outcomes are unaffected.

Observe moreover, that because of the signal invariance of first period excess demand functions for commodities and assets, the number of linearly independent first period constraints in any individual programming problem reduces to $(1 + S)$.

To circumvent the discontinuity in the excess demand map due to the shrinking of first period budget constraints, we restrict the domain of noninformative commodity prices to the set $\mathbf{P}^* \equiv \{\mathbf{p} \in \mathbf{P} \,|\, \mathbf{p}_1(s) \text{ is } s - \text{invariant and } p_{1,1}(1) = 1\}$, which is a smooth manifold without boundary, diffeomorphic to an open subset of $\mathbf{R}^{L+SL-1}$. The maps $z\left(p^*; \omega^h, \Sigma^h\right) \equiv \sum_{h \in \mathbf{H}} \left(z_1^h, z_2^h\right)\left(p^*; \omega^h, \Sigma^h\right)$ and $b^h\left(p^*; \omega^h, \Sigma^h\right)$ are smooth on the price domain $\mathbf{P}^*$.

**Lemma 12** *There is a open and dense set of full Lebesgue measure $\mathbf{\Omega}' \subset \mathbf{\Omega}$ such that, at fully noninformative rational expectations prices, every economy $\omega \in \mathbf{\Omega}'$ is regular.*

**Proof:** The proof is based on a transversality argument that exploits the signal-invariance restrictions the market clearing conditions satisfy when first period equilibrium prices are fully noninformative. Let $b^h\left(\theta^h\right)$ the net demand for bonds of household $h$ at joint signal $\boldsymbol{\theta}$. Recall that the bonds are the only instrument that allow households to redistribute their income across time. Since the state of nature occurring in the second period is still uncertain when the position on the financial market is to be closed, any asset function contains all the information available to any optimizing household. When rational expectations equilibrium prices are fully non revealing, they satisfy the $J \equiv \sum_{h \in \mathbf{H}} \left(\Theta^h - 1\right)$ extra conditions on bond signal-invariance, $b^h(\theta^h) - b^h(\theta'^h) = 0$, all $\theta'^h \in \mathbf{\Theta}^h$, $\theta^h \neq \theta'^h$, all $h \in \mathbf{H}$, and none of the individual private signals $\theta^h$ is disclosed. This also means that no matter the signal actually received, any household ends up with transferring the same negative or positive amount of money income to the second period. The system of equilibrium conditions of a fully non revealing economy is then

$$\begin{aligned} z\left(p^{*};\omega^{h},\Sigma^{h}\right) &= \mathbf{0}, && (L-1)\,(S+1)\ \textit{equations}, \\ \sum_{h\in\mathbf{H}} b^{h}(\theta^{h}) &= 0, && 1\ \textit{equation}, \\ b^{h}(\theta^{h}) - b^{h}(\theta'^{\,h}) &= 0, && J\ \textit{equations}, \\ \forall\theta'^{\,h} &\in \mathbf{\Theta}^{h},\ \theta^{h}\neq\theta'^{\,h},\ \forall h\in\mathbf{H}, \end{aligned}$$

We want to show that the smooth map $g : \mathbf{P}^{*}\times\mathbf{\Omega}\rightarrow\mathbf{R}^{L+J+(L-1)S}$, is transversal to zero, $g \pitchfork 0$. The map $g$ is defined as

$$g\left(\mathbf{p}^{*};\boldsymbol{\omega}^{h},\Sigma^{h}\right)\equiv\left(z\left(p^{*};\omega^{h},\Sigma^{h}\right),\sum_{h\in\mathbf{H}}b^{h}(\theta^{h}),(b^{h}(\theta^{h})-b^{h}(\theta'^{\,h}))_{\theta^{h}\in\mathbf{\Theta}^{h},h\in\mathbf{H}}\right)=\mathbf{0},$$

where $\mathbf{p}^{*}\equiv\left(\mathbf{p}_{1}^{*},\mathbf{p}_{2}^{*}(s)_{s\in\mathbf{S}}\right)$, $\boldsymbol{\omega}^{h}\equiv\left(\boldsymbol{\omega}_{1},\boldsymbol{\omega}_{2}(s)_{s\in\mathbf{S}}\right)$ and by Walras law $z$ is the aggregate excess demand function for all of first and the second period commodities but first period commodity one and second period commodity one in any state. The proof relies on the perturbation of the initial endowments. Observe that in standard models of sequential economies with an asset market for a transversality argument to hold, the endowment of a single, arbitrary household is typically perturbed. Here, the signal invariance constraints together with the measurability conditions on first period excess demand for commodities and assets, restrict first period endowments to a vector space of lower dimension than the consumption set. Hence standard perturbation techniques do not suffice. We proceed by identifying a matrix $\tilde{\boldsymbol{\omega}}$ of dimension $(L+S(L-1)+J)\times(HL(S+1))$. Its columns are $(L+S(L-1)+J)$-dimensional vectors defining the directions of $(HL(S+1))$ endowment perturbations: the derivative of $g$ with respect to one column of the matrix $\tilde{\boldsymbol{\omega}}$, is a directional derivative of $g$ in the direction identified by the column. This is the perturbation technique proposed by Pietra and Siconolfi [19].

We distinguish among three distinct directions of endowment perturbations: $\tilde{\boldsymbol{\omega}}\,(1,\ell)_{\ell>1}$ and $\tilde{\boldsymbol{\omega}}\left(b^{h}\left(\theta^{h}\right)\right)_{\theta^{h}\in\mathbf{\Theta}^{h},h\in\mathbf{H}}$ which refer to first period excess demand function for commodities other than commodity one and for bonds, and $\tilde{\boldsymbol{\omega}}\,(2,\ell,s)_{\ell>1,s\in\mathbf{S}}$ which refer to second period excess demand function for commodities other than commodity one:

$$\tilde{\boldsymbol{\omega}}\equiv\left[\tilde{\boldsymbol{\omega}}\,(1,\ell)_{\ell>1}\qquad\tilde{\boldsymbol{\omega}}\left(b^{h}\left(\theta^{h}\right)\right)_{\theta^{h}\in\mathbf{\Theta}^{h},h\in\mathbf{H}}\qquad\tilde{\boldsymbol{\omega}}\,(2,\ell,s)_{\ell>1,s\in\mathbf{S}}\right].$$

We want that the directional derivatives of the map $g$ with respect to the columns of the matrix $\tilde{\boldsymbol{\omega}}$ satisfy the following condition:

$$D_{\tilde{\boldsymbol{\omega}}}\ g=\mathbf{I},$$

where $\mathbf{I}$ is an identity matrix.

We perturb *every household* $h$'s endowment to induce full rank perturbations of both first period aggregate excess demand function $z_{1}\left(p^{*};\omega^{h},\Sigma^{h}\right)\equiv$

$\sum_{h\in\mathbf{H}}\left(z_1^h\right)\left(p^*;\omega^h,\Sigma^h\right)$, and excess demand for bonds $b^h\left(\theta^h\right)$.

We perturb *a single household* $h=1$'s endowment to induce full rank perturbations of second period aggregate excess demand function $z_2\left(p^*;\omega^h,\Sigma^h\right)\equiv\sum_{h\in\mathbf{H}}\left(z_2^h\right)\left(p^*;\omega^h,\Sigma^h\right)$.

Consider the column indexed by $\tilde{\boldsymbol{\omega}}\left(1,\ell\right)_{\ell>1}$ and an arbitrary household $h$. If his first period endowment of commodity $\ell$, with $\ell\neq 1$ is increased by one unit, $\triangle\ \omega_{1,\ell}^h=1$, and his first period endowment of commodity one is decreased by as much as the price of commodity $\ell$, i.e. $\triangle\ \omega_{1,1}^h=-p_{1,\ell}$, his income remains unaltered but his excess demand for first period commodities varies of $\triangle\ \mathbf{z}_1^h=\left(-p_{1,\ell},0,0,\ldots,1,0\right)$. the column $\tilde{\boldsymbol{\omega}}\left(1,\ell\right)_{\ell>1}$ of matrix $\tilde{\boldsymbol{\omega}}$ has all entries equal to zero, but $-p_{1,\ell}=\triangle\ \boldsymbol{\omega}_{1,1}$ and $1=\triangle\boldsymbol{\omega}_{1,\ell}$ for all $\ell\in\mathbf{L}_1$, $\ell\neq 1$. Therefore, the directional derivative of first period aggregate excess demand function $z_1\left(p^*;\omega^h,\Sigma^h\right)$ with respect to $\tilde{\boldsymbol{\omega}}\left(1,\ell\right)$ is $D_{\tilde{\boldsymbol{\omega}}(1,\ell)}\ z_{1,\ell}\left(p^*;\omega^h,\Sigma^h\right)=\mathbf{I}_{L-1}$.

Consider now every excess demand for bonds $b^h\left(\theta^h\right)_{\theta^h\in\boldsymbol{\Theta}^h}$ and the columns of matrix $\tilde{\boldsymbol{\omega}}$ indexed by $\tilde{\boldsymbol{\omega}}\left(b^h\left(\theta^h\right)\right)_{\theta^h\in\boldsymbol{\Theta}^h,h\in\mathbf{H}}$. If in every state $s\in\mathbf{S}$ we decrease every household $h$'s second period endowment of commodity $\ell=1$ by the bond payoff, $-(1+r)$, i.e. $\triangle\ \omega_{2,1}^h(s)=-(1+r)$, their income will remain unchanged provided that their demand for bonds $b^h\left(\theta^h\right)$ be increased by one unit, $\triangle\ b^h\left(\theta^h\right)=1$ and first period endowment of commodity $\ell=1$ be increased by one unit as well, $\triangle\ \omega_{1,1}^h=1$. Observe how the budget constraints of Eq.(8) of the *ex-ante modified economy* allow to independently perturb *all* of the $\Theta^h$ individual asset trading $b^h\left(\theta^h\right)$.

Consider finally the columns $\tilde{\boldsymbol{\omega}}\left(2,\ell,s\right)_{\ell>1,s\in\mathbf{S}}$ of matrix $\tilde{\boldsymbol{\omega}}$. If we increase $h=1$'s second period endowment of commodity $\ell$, with $\ell\neq 1$, in state $s$ by one unit, $\triangle\ \omega_{2,\ell}^1(s)=1$, and we decrease by its the price, $p_{2,\ell}(s)$, $h=1$'s endowment of commodity $\ell=1$ in the same state $s$, i.e. $\triangle\omega_{2,1}^1(s)=-p_{2,\ell}(s)$, the column $\tilde{\boldsymbol{\omega}}\left(1,\ell,s\right)_{\ell>1,s\in\mathbf{S}}$ of matrix $\tilde{\boldsymbol{\omega}}$ has all entries equal to zero, but the entry $\triangle\ \omega_{2,\ell}^1(s)=1$ and the entry $\triangle\ \omega_{2,1}^1(s)=-p_{2,\ell}(s)$.

The three types of endowment perturbation above guarantee that $D_{\tilde{\boldsymbol{\omega}}}\ g=\mathbf{I}$. Hence, all the equilibrium equations of a fully non revealing economy are linearly independent and the map $g$ is transversal to zero, $g\pitchfork 0$. By the transversality theorem there is a dense, full Lebesgue measure subset $\boldsymbol{\Omega}'\subset\boldsymbol{\Omega}$ of initial endowments, such that for each aggregate endowment vector $\boldsymbol{\omega}\in\boldsymbol{\Omega}'$, $g_\omega\pitchfork 0$. The boundary conditions satisfied by the aggregate excess demand function ensure that the subset $\boldsymbol{\Omega}'$ is open. Therefore, by the implicit function theorem, generically in endowment space, each fully non revealing economy $\boldsymbol{\omega}\in\boldsymbol{\Omega}'$ is regular. $\square$

**Step 2:** We can now establish the result of Proposition 10.

In Step 1 we have shown that the map $g_\omega:\mathbf{P}\to\mathbf{R}^{L+J+(L-1)S}$ is generically transversal to zero, $g_\omega\pitchfork 0$. Given that $g_\omega$ smoothly maps the smooth manifold

$\mathbf{P}^*$ , the commodity price set, of dimension $(L+SL-1)$ into a smooth manifold of dimension $(L+S(L-1)+J)$, $g_\omega^{-1}(0)$ is a smooth submanifold of dimension $(S-1-J) \equiv (S-1) - \sum_{h\in\mathbf{H}} (\Theta^h - 1)$. $\square$

# 5 Partially revealing ree

A rational expectations equilibrium is partially revealing when first period prices guarantee that at least one (private) signal is not disclosed at equilibrium so that households, not having the possibility of completing their knowledge refinement, remain asymmetrically informed at equilibrium.

Partially revealing economies decompose into fully informative and fully non-informative subeconomies.

**Definition 13** *A rational expectations equilibrium for an economy $u \in \acute{U}$, $\acute{U} \subset \mathbf{U}$, is partially revealing if first period commodity prices do not distinguish among states $s, s' \in \hat{\mathbf{S}}$, of a subset $\hat{\mathbf{S}} \subset \mathbf{S}$, while are injective in all the remaining states that belong to the set $\bar{S} = \mathbf{S} \backslash \hat{\mathbf{S}}$:*

$$\mathbf{p}_1(\hat{s}) = \mathbf{p}_1(\hat{s}') \qquad \hat{s}, \hat{s}' \in \hat{\mathbf{S}},\ \hat{s} \neq \hat{s}',$$

*and*

$$\mathbf{p}_1(\bar{s}) \neq \mathbf{p}_1(\bar{s}') \qquad \bar{s}, \bar{s}' \in \bar{\mathbf{S}},\ \bar{s} \neq \bar{s}'.$$

The configurations of partially revealing rational expectations equilibria are many. In the simplest case, first period prices do not distinguish between two signals $\hat{\theta}^h, \hat{\theta}'^h$ of a subset $\hat{\mathbf{\Theta}}^h \subset \mathbf{\Theta}^h$ of cardinality $\hat{\Theta}^h = 2$ of just one individual $h \in \mathbf{H}$, while they convey entirely the private information of all the other individuals $k \neq h$, $k \in \mathbf{H}$. In this case, first period partially revealing prices may be equivalently defined as:

$$\mathbf{p}_0(\hat{\theta}^h, \theta^{-h}) = \mathbf{p}_0(\hat{\theta}'^h, \theta^{-h}), \qquad \hat{\theta}^h, \hat{\theta}'^h \in \hat{\mathbf{\Theta}}^h, \quad \hat{\theta}^h \neq \hat{\theta}'^h$$

and

$$\mathbf{p}_0(\bar{\theta}^h, \theta^{-h}) \neq \mathbf{p}_0(\bar{\theta}'^h, \theta^{-h}) \qquad \bar{\theta}^h, \bar{\theta}'^h \in \bar{\mathbf{\Theta}}^h, \quad \bar{\theta}^h \neq \bar{\theta}'^h.$$

where $\bar{\mathbf{\Theta}}^h = \mathbf{\Theta}^h \backslash \hat{\mathbf{\Theta}}^h$ : first period partially revealing prices are $\hat{\theta}^h$-invariant and, at the same time, they are injective both, in all the remaining signals $\bar{\theta}^h \in \bar{\mathbf{\Theta}}^h$ household $h$ may receive and in all signals $\theta^{-h} \in \mathbf{\Theta}^{-h}$ where $\theta^{-h}$ is an $(H-1)$-dimensional array of signals received by individuals $k \in \mathbf{H}$, $k \neq h$.

Note that when $\hat{\Theta}^h = 2$ , household $h$ is the only individual who, after observing the price realization $\mathbf{p}_1(\hat{\theta}^h, \theta^{-h})$, has the possibility to disclose the uncertainty on the state of the world occurring in the second period as he joins his own private information with the information of all the other individuals conveyed by first period equilibrium prices. Households $k \in \mathbf{H}$, $k \neq h$ will not be able to perfect their knowledge at equilibrium.

Evidently, when the households whose private information is not revealed by first period prices are more than one, nobody will discover in the first period which states realizes in the second.

### 5.1 Existence of partially revealing ree

As when analyzing fully revealing rational expectations equilibria, we consider the case in which, first period consumption plans of (at least) an individual $h = 1$ in three subeconomies $\bar{s}$, $\bar{s}'$, and $\hat{s}$, where $\bar{s}, \bar{s}' \in \bar{\mathbf{S}}$ and $\hat{s} \in \hat{\mathbf{S}}$, are distinct: $\mathbf{x}_1^1(\bar{s}) \neq \mathbf{x}_1^1(\bar{s}') \neq \mathbf{x}_1^1(\hat{s})$. In this case any perturbation of $h = 1$'s Bernoulli utility functions as described in sections 2.2.1 and 3.1 may be used to show the regularity of the three subeconomies. For an argument similar to that of sections 3.1 and 4.1 first period equilibrium prices are locally unique so that: $\mathbf{p}_1(\bar{s}) \neq \mathbf{p}_1(\bar{s}') \neq \mathbf{p}_1(\hat{s})$. Hence partially revealing rational expectations equilibria exist.

**Proposition 14** *There is an open and dense set of economies $\acute{\acute{U}} \subset \mathbf{U}$, such that for each $u \in \acute{\acute{U}}$ every subeconomy $\hat{s} \in \hat{\mathbf{S}}$ and $\bar{s} \in \bar{\mathbf{S}}$ is regular. Hence partially revealing rational expectations equilibria exist.*

**Proof:** The argument follows from Proposition 7 and Lemma 12.

### 5.2 The set of partially revealing ree

**Proposition 15** *There is a (relatively) open, dense set of full Lebesgue measure $\mathbf{\Omega}' \subset \mathbf{\Omega}$, such that for every economy $\omega \in \mathbf{\Omega}'$ the set of partially revealing rational expectations equilibria contains a submanifold of dimension*

$$\left(\hat{S}-1\right)-\sum_{h\in\mathbf{H}}\left(\hat{\Theta}^h-1\right),$$

*where $\hat{\mathbf{\Theta}}^h \subset \mathbf{\Theta}^h$ are the subsets of private signals which are not disclosed at equilibrium.*

**Proof:** The argument follows from Proposition 10.

## 6 Monetary policy

We will now extend the model presented above by including a *central bank*, which essentially acts as a social planner, and *paper money.* Money has no effect on households' utility and serves only as a medium of exchange.

**Assumption 16** *: Barter and the direct consumption of one's own endowment are not allowed.*

Assumption 16 guarantees that any amount of any good that is consumed, is before purchased with a well - defined amount of money.

There is no exogenous supply of money and to explain its creation and destruction we think of the model of the ideal bank of Wicksell [27] in which the bank acts as an intermediary. For simplicity, we assume that there are no charges for the services it provides.

Every household, at the beginning of every time period, in order to trade in the market, sells an asset called *money* to the bank and receives some paper notes in exchange. The asset money does not pay any interest.

The creation of money is then due to the fact that the bank buys assets which are not means of payment and pays them with an asset which is a mean of payment. The bank creates new money, monetizing activities which are not mediums of exchange.

Since money does not pay any interest as opposed to the asset $b$, *nobody is willing to hold money as a store of value* (Magill and Quinzii [15]) and since the bank is just an intermediary and the possibility of *default is excluded*, by the end of the first period, every household gives the liquidity he received back, i.e. he buys back from the bank the asset $m_1^h$ sold at the beginning of the period with the money balances provided by the sale of the endowment received in the first period. If the money value of first period endowment $\mathbf{p}_1 \cdot \boldsymbol{\omega}_1^h$, is different from the money value of first period consumption $\mathbf{p}_1 \cdot \mathbf{x}_1^h$, the household trades the asset $b$ to readjust his position. The demand for money of every household $h \in \mathbf{H}$ in the first period is hence defined by

$$m_1^h = \mathbf{p}_1 \cdot \mathbf{x}_1^h = \mathbf{p}_1 \cdot \boldsymbol{\omega}_1^h + b^h.$$

At the beginning of the second period, according to his position on the bond market and to his expectations on the proceedings from the sale of his products, every household to finance his consumption plan, demands money

$$m_2^h(s) = \mathbf{p}_2(s) \cdot \mathbf{x}_2^h(s) = \mathbf{p}_2(s) \cdot \boldsymbol{\omega}_2^h(s) - (1+r)b^h, \quad s = 1, \ldots, S.$$

Households are free to use their money in the way they like, but holding money does not provide them any utility *per se*, therefore by the end of the period every household closes his position with the bank so that $m_2^h(s) = 0$ for all $h \in \mathbf{H}$, $s \in \mathbf{S}$. *Any money creation is then followed by an automatic money destruction.*

The quantity of money that bank makes available for the households to transact defines the *monetary policy* pursued.

At the beginning of the first period, before the private signals are sent to the households, the bank selects, according to a *social welfare criterion* discussed in section 8, the monetary policy it will implement and makes it public.

**Assumption 17** *The bank announces the monetary policy at the beginning of the first period, before trade occurs and before households receive any private signal.*

The bank provides liquidity to the economy following a rule which essentially draws on the logic of the quantity theory.

**Assumption 18** *In the first period, the bank injects in the economy,*

$$m_1 = \mathbf{p}_1 \cdot \boldsymbol{\omega}_1^h, \tag{9}$$

*fixed in order to ensure that the price of the bond be equal to* $q = 1$.
*In the second period, the bank satisfies the demands for money of the households suppling*

$$m_2(s) = \mathbf{p}_2(s) \cdot \boldsymbol{\omega}_2^h(s), \qquad s \in \mathbf{S}. \tag{10}$$

Note that Assumptions 16 and 18 require that each household sells all of his endowment for paper money. In models of monetary economies this is a conventional assumption which has been motivated in many ways: to show that money allows specialization as, when households are highly specialized, they need a medium of exchange to trade in the market of commodities; to treat the issue of occupational choice as part of individual demand decision and with an informational motivation.

# 7 Monetary policy and information

Assumptions 16 and 18 ensure that the meeting between the money supplied and the aggregate demand for money well defines the price level of money in terms of the price of all the commodities exchanged in the economy solving for both the *nominal* and the *real* for *indeterminacy* in prices that characterizes monetary economies with an incomplete asset market.

As a matter of fact, it is a well known result that in two-period nominal asset sequential economies with an incomplete financial market, competitive equilibria generically display $(S+1)$ degrees of indeterminacy in prices and allocations. In particular, $(S-1)$ degrees of indeterminacy, affecting the purchasing power of the asset payoffs, modify the asset span and may translate into a real indeterminacy of equilibrium prices and excess demands, while the 2 remaining degrees of indeterminacy are purely nominal. Since nominal indeterminacy does not affect neither consumption nor portfolio choices, we offset the nominal redundancies both, by normalizing the asset price to one, $q = 1$ with an ad hoc first period money supply, $m_1 = \alpha$, $\alpha \in \mathbf{R}$, where $\alpha$ is kept constant and by means of second period, state one money supply $m_2(1) = \beta$, $\beta \in \mathbf{R}$, where $\beta$ is kept constant. Every monetary policy vector $\mathbf{m} = (m_1, m_2(1), \ldots, m_2(S))$ places then a constraint on equilibrium prices and if households refine their knowledge by observing equilibrium prices, it is clear that *the choice of monetary policy affects their information* and hence their actions. Magill and Quinzii [15] showed that the monetary policy implemented in a monetary economy with an incomplete asset market entails real effect on the allocations of resources at equilibrium. Here we extend the result of Magill and Quinzii [15] to economies with asymmetric information and show how monetary policy may select the parametrization of the submanifolds of equilibria to determine the degree of information revelation of rational expectations prices.

## 7.1 Fully non revealing monetary policy

Section (4) showed that when rational expectations equilibrium prices are fully non revealing, they satisfy the $J \equiv \sum_{h\in\mathbf{H}} \left(\Theta^h - 1\right)$ extra conditions on bond signal-invariance, $b^h(\theta^h) - b^h(\theta'^h) = 0$, all $\theta'^h \in \mathbf{\Theta}^h$, $\theta^h \neq \theta'^h$, all $h \in \mathbf{H}$, and none of the individual private signals $\theta^h$ is disclosed. This also means that no matter the signal actually received, any household ends up with transferring the same negative or positive amount of money income to the second period. The system of equilibrium conditions of a fully non revealing economy without money is then

$$\begin{array}{rcll} z\left(p^*;\omega^h,\Sigma^h\right) &=& \mathbf{0}, & (L-1)(S+1)\ \textit{equations}, \\ \sum\limits_{h\in\mathbf{H}} b^h(\theta^h) &=& 0, & 1\ \textit{equation}, \\ b^h(\theta^h) - b^h(\theta'^h) &=& 0, & J\ \textit{equations}, \\ \forall\theta'^h &\in& \mathbf{\Theta}^h,\ \theta^h \neq \theta'^h,\ \forall h \in \mathbf{H}. & \end{array}$$

Proposition (10) showed that except for a nongeneric subset of endowments, the set of fully non revealing rational expectations equilibria contains a submanifold of dimension $(S-1) - \sum_{h\in\mathbf{H}} \left(\Theta^h - 1\right)$. When monetary policy parametrizes both the nominal and the real indeterminacy, i.e. a smooth manifold of dimension $(S+1) - \sum_{h\in\mathbf{H}} \left(\Theta^h - 1\right)$, the system of equilibrium conditions becomes

$$\begin{array}{rcll} z\left(p^*;\omega^h,\Sigma^h\right) &=& \mathbf{0}, & (L-1)(S+1)\quad \textit{equations}, \\ \sum\limits_{h\in\mathbf{H}} b^h(\theta^h) &=& 0, & 1\ \textit{equation}, \\ m_1 &=& \mathbf{p}_1 \cdot \boldsymbol{\omega}_1^h & \\ m_2(s) &=& \mathbf{p}_2(s) \cdot \boldsymbol{\omega}_2^h(s), & s = 1,\ldots,S\quad \textit{equations.} \end{array}$$

A monetary policy is fully non revealing if it guarantees that equilibrium prices satisfy the constraint on signal-invariance of the net demand for bonds. To identify fully non revealing monetary policy vectors, we have then to find out which prices ensure the signal-invariance of individual asset trading. We are therefore looking for those prices which make private information useless. Said in other terms, we are looking for those prices which are "compatible" with every possible configuration of aggregate private information so that whatever signal an household may receive, being an optimizing price-taker, he will end up with taking the same consumption and savings decision. A fully non revealing monetary policy vector has therefore to satisfy the following market clearing conditions:

$$\begin{array}{rcll} z\left(p^*;\omega^h,\Sigma^h\right) & = & \mathbf{0}, & (L-1)\,(S+1)\ \textit{equations}, \\ \sum_{h\in\mathbf{H}} b^h(\theta^h) & = & 0, & 1\ \textit{equation}, \\ b^h(\theta^h)-b^h(\theta'^h) & = & 0, & J\ \textit{equations}, \\ \forall\theta'^h & \in & \mathbf{\Theta}^h,\ \theta^h\neq\theta'^h,\ \forall h\in\mathbf{H}, & \\ m_1 & = & \alpha, & \alpha\ \textit{constant} \\ m_2(1) & = & \beta, & \beta\ \textit{constant} \\ m_2(s) & = & \boldsymbol{\gamma}\in\mathbf{R}, & s=J+2,\ldots,S. \end{array} \tag{11}$$

We know that monetary policy is a vector of dimension $(S+1)$ : the components $m_1$ and $m_2(1)$ are used to offset the nominal redundancies; the value of other $(J)\equiv\sum_{h\in\mathbf{H}}\left(\Theta^h-1\right)$ components is not under control of the bank but it is fixed by the constraints on the non-revelation of private information; to the remaining $(S-J-2)$ components the bank is free to assign whatever value, $m_2(s)=\boldsymbol{\gamma}\in\mathbf{R}$, $s=J+2,\ldots,S$.

We have therefore to identify the value that has be exogenously assigned $J$ in order to guarantee non revelation. We solve system (11), we estimate the equilibrium commodities for $J$ states of nature and we substitute their values into equations (10).

## 7.2 Partially revealing monetary policy

Section (5) showed that a rational expectations equilibrium is partially revealing when first period prices guarantee that at least one (private) signal is not disclosed at equilibrium so that households, not having the possibility of completing their knowledge refinement, remain asymmetrically informed at equilibrium. Proposition (15) proved that except for a nongeneric subset of endowments, the set of fully non revealing rational expectations equilibria is contains a submanifold of dimension $\left(\hat{S}-1\right)-\sum_{k\in\mathbf{K}\subset\mathbf{H}}\left(\hat{\Theta}^k-1\right)$.

At a partially revealing rational expectations equilibrium therefore only *some* individual net demands for bonds are signal-invariant. A monetary policy is partially revealing if it guarantees that equilibrium prices satisfy the constraint on signal-invariance of the net demand for bonds of *some* informed households.

A partially revealing monetary policy vector has to satisfy the following market clearing conditions:

$$\begin{aligned}
z\left(p^*;\omega^h,\Sigma^h\right) &= \mathbf{0}, && (L-1)\,(S+1)\ \textit{equations}, \qquad (12)\\
\sum_{h\in\mathbf{H}} b^h(\theta^h) &= 0, && 1\ \textit{equation},\\
b^{\hat{h}}(\hat{\theta}_1) - b^{\hat{h}}(\hat{\theta}_k) &= 0, && k=1,\ldots,\hat{\Theta}^{\hat{h}}\ \ \textit{and}\ \ \hat{h}=1,\ldots,\hat{H},\\
& && K\ \textit{equations},\\
m_1 &= \alpha, && \alpha\ \textit{constant}\\
m_2(1) &= \beta, && \beta\ \textit{constant}\\
m_2(s) &= \boldsymbol{\delta}\in\mathbf{R}, && s=K+2,\ldots,S,
\end{aligned}$$

where $\hat{\mathbf{H}} \subset \mathbf{H}$ is the subset of households whose private information is not disclosed and $\hat{\boldsymbol{\Theta}}^{\hat{h}} \subset \Theta^{\hat{h}}$ is the subset of their signals that are not conveyed by first period prices. Monetary policy is a vector of dimension $(S+1)$ : the components $m_1$ and $m_2(1)$ are used to offset the nominal redundancies; the value of other $K \equiv \sum_{h\in\mathbf{H}}\left(\hat{\Theta}^h - 1\right)$ components is not under control of the bank but it is fixed by the constraints on the non-revelation of private information; to the remaining $(S-K-2)$ components the bank is free to assign whatever value, $m_2(s) = \boldsymbol{\delta} \in \mathbf{R}$, $s = K+2,\ldots,S$. As before, to identify the value that has to be exogenously assigned to $K$ monetary policy components in order to guarantee partial non revelation. We solve system (12), we estimate the equilibrium commodities for $K$ states of nature and we substitute their values into equations (10).

# 8 Welfare

It is a well known result that when the asset market is incomplete and the financial structure is taken as given, equilibrium is typically ex - ante inefficient; hence, in general, the market does not make an optimal use of the available resources and an exogenous intervention may be required. This is particularly true when in the economy households are asymmetrically informed, because information may be another source of welfare loss; as pointed out by Hirshleifer [12], the role of information is ambiguous: when a complete set of signal - contingent markets and state - contingent financial markets are missing, fully revealing rational expectations equilibrium prices entail a positive effect on allocations and a negative effect on risk sharing; on one side they may transmit too much information to the economy reducing asset trading and hence risk sharing among households; on the other hand, it is true that just one financial asset is enough to ensure the ex - post Pareto optimality.

Here the intervention of the bank affects individual welfare in two ways: by fixing the aggregate money supply, not only the bank lifts the indeterminacy in prices and in equilibrium allocations, but it also affects households' welfare by determining to which extent prices aggregate and convey information.

We assume that the goal of the bank consists in implementing the monetary policy that leads the economy towards socially preferred equilibria. In particular, the bank selects the monetary policy vector that ensures the maximization of the following *expected social welfare function (*this requires measurability and comparability of households' utility functions; under assumption 3 utility functions are partially measurable, we hypothesize comparability):

$$W_1 = \sum_{s \in \mathbf{S}} \pi_s \sum_{h \in \mathbf{H}} \alpha^h \, v^h \left( \mathbf{x}^h(s) \right), \tag{13}$$

this calls for a judgement on how to aggregate individual utilities, so $\alpha^h$ are the weights set in order to keep into account the degree of inequality of utility values; the bank is willing to accept a decrease in the utility of some types of households, only if there is a larger increase in the utility of some other groups of households; we assume that $\sum_{h \in \mathbf{H}} \alpha^h = 1$; moreover, $v^h \left( \mathbf{x}^h(s) \right) = u_1(\mathbf{x}_1^h) + u_s^h(\mathbf{x}_s^h)$ $s = 1, \ldots, S$ are the values of *ex - post* Bernoulli state - dependent cardinal utility functions.

Under assumption 17, monetary policy is announced at the beginning of the first period, when the choice of nature on the state of the world prevailing in the second period is completely unknown to the economy; we assume that the expectations of the bank, $\pi_s$, are the common priors of the economy before the private signals are sent to the households. The bank, therefore, does not receive any kind of private information and the probability it attaches to any possible future outcome depends on the commonly observed (relative) frequency of the event itself.

The bank decides whether preventing or partially blocking the revelation of private information in the economy only if the expected social welfare function it maximizes displays a higher value than when a fully revealing monetary policy is implemented.

# 9 A numerical example

Consider a two-period pure exchange economy with three types $h = 0, 1, 2$ of households; there is a continuum of households for each type. Individual types differ in endowment and information; they trade a single nominal bond $b$ and three types of commodities $\ell = 0, 1, 2$; paper money is the medium of exchange.

The initial endowment of each individual consists of a single type of commodity. households $h = 0$ receive $\omega_1^0 = 50$, a deterministic amount of good $\ell_0$, in the first period; households $h = 1$ receive either a *high* or a *low* amount of good $\ell_1$ in the second period, $\omega^1 = (100, 200)$; also households $h = 2's$ second period endowment of commodity $\ell_2$ is random: $\omega^2 = (200, 400)$; households $h = 1, 2$ receive nothing in the first period and households $h = 0$ do not receive anything in the second period.

households $h = 1, 2$ in the first period receive a signal $\theta_\omega^h \in \mathbf{\Theta}^h = \left\{ \theta_1^h, \theta_2^h \right\}$ which is correlated with their endowment. Households $h = 0$ do not receive any private information. The state space is then defined as: $S \equiv \mathbf{\Theta}^1 \times \mathbf{\Theta}^2$; its cardinality is $S = 4$. The probability households $h = 1$ attach to their signals are $\pi^1\left(\theta_1^1\right) = 0.3$ and $\pi^1\left(\theta_2^1\right) = 0.7$; the probability assessed by households $h = 2$ for their signals are $\pi^2\left(\theta_1^2\right) = 0.4$ and $\pi^2\left(\theta_2^2\right) = 0.6$. Table 1 shows households' ex-post beliefs for every $s \in S$.

Table 1: Individuals' expectations conditional on private information

| state, $\omega^1(s)$, $\omega^2(s)$, | $h=0$ $\pi^0(s)$ | $h=1$ $\pi^1(s\|\theta_1^1)$ | $\pi^1(s\|\theta_2^1)$ | $h=2$ $\pi^2(s\|\theta_1^2)$ | $\pi^2(s\|\theta_2^2)$ |
|---|---|---|---|---|---|
| s=1: $(100, 200)$ | 0.12 | 0.3 | 0 | 0.4 | 0 |
| s=2: $(100, 400)$ | 0.18 | 0.7 | 0 | 0 | 0.4 |
| s=3: $(200, 200)$ | 0.28 | 0 | 0.3 | 0.6 | 0 |
| s=4: $(200, 400)$ | 0.42 | 0 | 0.7 | 0 | 0.6 |

Table 2 summarizes the relations among true states and signals:

Table 2: Relation between private signals and the true state $s^*$

| *individual* | Signal | signal's probability | True state $s^*=1$ | $s^*=2$ | $s^*=3$ | $s^*=4$ |
|---|---|---|---|---|---|---|
| h=1 | $\theta_1^1$ | $\pi^1(\theta_1^1) = 0.4$ | $\times$ | $\times$ | | |
| h=1 | $\theta_2^1$ | $\pi^1(\theta_2^1) = 0.6$ | | | $\times$ | $\times$ |
| h=2 | $\theta_1^2$ | $\pi^2(\theta_1^2) = 0.3$ | $\times$ | | $\times$ | |
| h=2 | $\theta_2^2$ | $\pi^2(\theta_2^2) = 0.7$ | | $\times$ | | $\times$ |

In the first period, households trade at price $q = 1$ a nominal asset $b$ which pays $(1 + r) = 1.04$ units of money in period 1 regardless of the prevailing state of the world. In the first period commodity $\ell_0$ is traded at price $p_{1,0}$ and in the second period there is a spot market where commodities $\ell_1$ and $\ell_2$ are exchanged at prices $p_{2,1,s}$ and $p_{2,2,s}$ with $s = 1, \ldots, 4$.

Households act rationally and solve the following ex-post planning problem:

$$\max_{x^h, b^h} \quad U^h(\mathbf{x}^h) = \sum_{s \in \boldsymbol{\sigma}_p^h} \pi^h(s|\boldsymbol{\sigma}_p^h) \left[\mu_1^h \ln x_1^h(\boldsymbol{\sigma}_p^h) + \mu_{2,1,s}^h \ln x_{2,1,s}^h + \mu_{2,2,s}^h \ln x_{2,2,s}^h\right],$$

subject to the budget constraint:

$$\begin{aligned}
\mathbf{p}_1(s) z_1^h(s) + b^h(s) &= 0, \quad s \in \boldsymbol{\sigma}_p^h, \\
\mathbf{p}_2(s) \cdot \mathbf{z}_2^h(s) - (1 + r) b^h(s) &= 0, \quad s \in \boldsymbol{\sigma}_p^h, \\
(\mathbf{z}_1^h, b^h) \ \textit{s-invariant in } \boldsymbol{\sigma}_p^h, &
\end{aligned}$$

Each individual $h's$ propensities to consume sum up to one, $\mu_1^h + \mu_{2,1,s}^h + \mu_{2,2,s}^h = 1$.
We assume that the utility parameters:
of $h = 0's$ are state-dependent, $\mu_1^0 = 0.35$, $\mu_1^0(s) = \{0.5,\ 0.4,\ 0.35,\ 0.55\}$;
of $h = 1's$ are state-independent, $\mu^1 = \{0.3\ ,\ 0.55\ ,\ 0.15\}$;
of $h = 2's$ are state-independent, $\mu^2 = \{0.25\ ,\ 0.5\ ,\ 0.25\}$.

## 9.1 Monetary policy

At the beginning of the first period, the central bank announces a money supply plan, a $(S+1)-$ dimensional vector $\mathbf{m} = (m_1, m_2(1), \ldots, m_2 4)$.
$m_1$ normalizes the price of the bond and $m_2(1)$ eliminates the other nominal redundancy and it is assigned value 1772; we can think that $m_2(1)$ sets the scale of money of the economy.
The values assigned to entries $m_2(s)$, $s = 2, 3, 4$, are increased exponentially at the rate of 10% in the range $(1000 \div 10800)$ so that we explored a grid of $17,576$ distinct monetary policies. Such a grid that can be imagined shaped as a cube see Fig. (1): since parameter $m_2(1)$ has just a nominal effect on the price level it is kept constant, parameters, $m_2(s)$, $s = 2, 3, 4$, which entail a real effect on equilibrium, have been divided by $m_2(1)$. Then, on the $x$-axis we may plot the ratio $\log \frac{m_2(2)}{m_2(1)}$, on the $y$-axis the ratio $\log \frac{m_2(3)}{m_2(1)}$ and on the $z$-axis $\log \frac{m_2(4)}{m_2(1)}$. We considered the logarithm of each ratio since the values assigned to money supply have been increased exponentially; this implies that, despite $\mathbf{M} \in \mathbf{R}^4_{++}$, negative values appear on the axis.

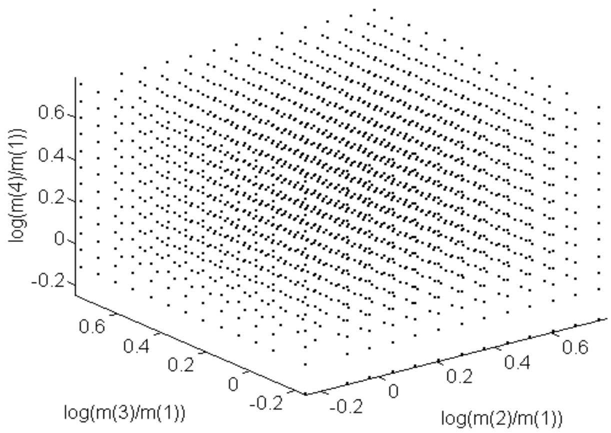


Figure 1: The monetary policy set.

## 9.2 Fully revealing monetary policy

When the bank decides not to prevent the disclosure of private information, the market clearing system of equation *in each subeconomy s* is:

$$\begin{aligned} \sum_{h \in \mathbf{H}} b^h(s) &= 0, \\ \sum_{h \in \mathbf{H}} \mathbf{x}_2^h(s) - \boldsymbol{\omega}_2(s) &= \mathbf{0}, \\ p_{2,1}(s)\omega_{2,1}(s) + p_{2,2}(s)\omega_{2,2}(s) &= 1772 \end{aligned} \tag{14}$$

The value assigned to money supply is not relevant in terms of distribution of resources at equilibrium in a fully revealing rational expectation equilibrium, i.e. monetary policy is ex-post neutral. System (14) has 3 equations and 3 unknowns: $p1(s)$, $p_{2,1}(s)$ and $p_{2,2}(s)$.

## 9.3 Partially revealing monetary policy

Let us assume that the private information of agents $h = 1$ be not disclosed by equilibrium prices so that all households but him cannot distinguish $s_1$ from $s_2$, i.e. $p_1(s_1) = p_1(s_2)$. To derive the monetary policy vector which ensures such an equilibrium we have to solve the following system of market clearing conditions:

$$\begin{aligned} \sum_{h \in \mathbf{H}} b^h(s) &= 0, \qquad\qquad s = 1, ..., 4 \\ b^1(\theta_1^1) &= b^1(\theta_2^1) \\ \sum_{h \in \mathbf{H}} \mathbf{x}_2^h(s) - \omega_2(s) &= \mathbf{0}, \qquad s = 1, ..., 4 \\ m_2(1) &= 1772 \\ m_2(2) &= \alpha \in \mathbf{R} \\ m_2(3) &= \beta \in \mathbf{R} \end{aligned} \tag{15}$$

System (15) has 12 equations and 12 unknowns: $(p_1(s), p_{2,1}(s), p_{2,2}(s))_{s \in \mathbf{S}}$. Observe that from equilibrium prices $\mathbf{p}_1^*(4), \mathbf{p}_2^*(4)$ the value of monetary policy in state 4 can be estimated according to the rule: $m_2(s) = \mathbf{p}_2(s) \cdot \boldsymbol{\omega}_2^h(s)$. In this case, 2 out of 4 components of a monetary policy vector are under the bank's discretions, while of the remaining 2, one is fixed by the non-revelation constraint and the other is not relevant. Note that the scalars $\alpha$ and $\beta$ define a plane which is the set of partially revealing monetary policies that ensure that at equilibrium $p_1(s_1) = p_1(s_2)$. See Fig. (2).

## 9.4 Fully non revealing monetary policy

To derive the monetary policy vector that ensures a fully non revealing rational expectations equilibrium, we have to solve the following system of market

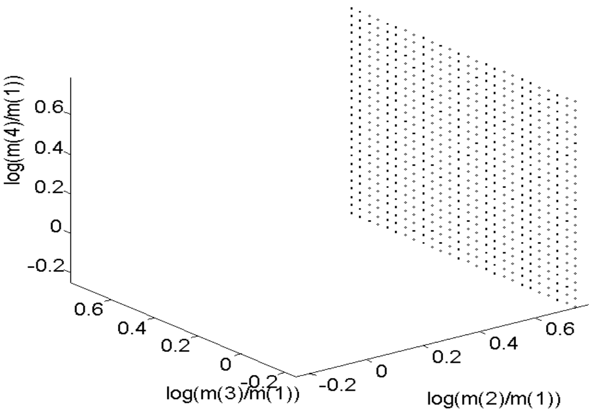


Figure 2: Partially revealing monetary policy set: s1=s2

clearing conditions:

$$\begin{aligned} \sum_{h\in\mathbf{H}} b^h(s) &= 0, \qquad\qquad s=1,...,4 \\ b^1(\theta_1^1) &= b^1(\theta_2^1) \\ b^2(\theta_1^2) &= b^2(\theta_2^2) \\ \sum_{h\in\mathbf{H}} \mathbf{x}_2^h(s)-\omega_2(s) &= \mathbf{0}, \qquad s=1,...,4 \\ m_2(1) &= 1772 \\ m_2(2) &= \alpha\in\mathbf{R} \end{aligned} \tag{16}$$

System (16) has 12 equations and 12 unknowns: $(p_1(s), p_{2,1}(s), p_{2,2}(s))_{s\in\mathbf{S}}$. Observe that from equilibrium prices $p_1^*(3), \mathbf{p}_2^*(3)$ and $p_1^*(4), \mathbf{p}_2^*(4)$ the value of monetary policy in states 3 and 4 can be estimated according to the rule: $m_2(s) = \mathbf{p}_2(s)\cdot\boldsymbol{\omega}_2^h(s)$. In this case just one out of 4 second period components of a monetary policy vector is under the bank's discretions, while of the remaining 3, two are fixed by the non-revelation constraints and the other is not relevant. Note that the scalar $\alpha$ defines a line which represents is the set of fully non revealing monetary policies that ensures that at equilibrium $p_1(s_1) = p_1(s_2) = p_1(s_3) = p_1(s_4)$. See Fig. (3).

## 9.5 Monetary policy and welfare

In our model the asset structure is fixed, there is always a single bond available, therefore with only one nominal asset with constant payoff for the transfer of revenue across time periods and realizations of uncertainty, households do

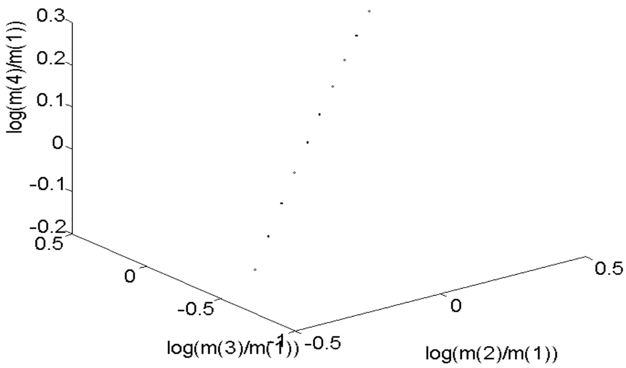


Figure 3: The fully non revealing monetary policy set

not have the possibility to insure against risk. The redistribution of wealth at equilibrium depends therefore on the enforced monetary policy.

Since equilibrium allocations are typically ex-ante inefficient, any improvement in one individual's welfare does not require a reduction in the welfare of some other individual.

**Households h=0**

Households of type $h = 0$ receive their endowment in the first period and in order to consume in the second period they transfer part of their income by means of the bond $b$. Hence, they are best off at those monetary policies that guarantee a "high" (in relative terms) purchasing power of their savings. Such condition is guaranteed by *fully revealing* monetary policies; recall that when prices pool and convey all the aggregate information of the economy, monetary policy is neutral, equilibrium allocations are ex-post Pareto optimal and households $h = 0$ had the possibility to get insured in the best way possible.

Households $h = 1, 2$ on the contrary, are worst off when the bank enforces a fully non revealing monetary policy injecting a lot of money in the economy. In this case, in fact, uncertainty prevents them from transferring their wealth as efficiently in the future and this results in a welfare loss.

**Households h=1,2**

Households $h = 1, 2$ get indebted in the first period therefore are best off at the highest inflationary policy the bank can implement; their welfare is also improved by the presence of uncertainty as they can share the risk they bear on the endowment they receive in the second period, with other households.

Households $h = 1, 2$ being privately informed, are better off at partially revealing equilibria as they can exploit their private information; as Hirshleifer

[12] pointed out, in a competitive economy, individual's choice would only negligibly affect equilibrium prices; hence, an informed individual can realize all the market value of his endowment in the states he alone knows will not occur and reallocate his income to purchase commodities in the states he knows are more likely; his utility will then be increased.

It is interesting to remark that households $h = 0$, who are endowed with the most preferred commodity in the economy, have their welfare affected by monetary policy very little; in any case they are worst off at fully revealing equilibria.

Households $h = 2$ are best off at a fully non revealing monetary policy and worst off when there is full revelation.

## 9.6 Welfare analysis

Given the potential conflict among households in terms of welfare, the bank has to decide which weights to assign to individual utilities in the social welfare function it wants to maximize; in Table 3 we report the results of numerical simulation run when the social welfare function considered is:

$$W_1 = \sum_{s \in \mathbf{S}} \pi_s \sum_{h \in \mathbf{H}} \alpha^h \, v^h \left( \mathbf{x}^h \right), \tag{17}$$

The probabilities $\pi_s$ used to compute expected values are the common, ex-ante estimates of the least informed individual: $\boldsymbol{\pi}^1 = \{(0.12\,,\,0.18\,,\,0.28\,,\,0.42)\}$.

Recall that weights $\sum_h \alpha^h = 1$. We have explored a grid of different values of the coefficient $\alpha^h$ the central bank assigns to every type of individual. The central bank process of heterogeneous preference aggregation is taken to be random and is manifested through its policymaking and the correspondent social outcomes. The bank may decide to eliminate individual uncertainty (limiting therefore risk sharing) and to make a certain group of households better off , i.e. households $h = 0$ by announcing a policy that ensures the full revelation of the economy's aggregate information, or it may allow some or complete uncertainty leaving insurance opportunities and enhancing the welfare of some households, i.e. $h = 1, 2$ but letting others incur in adverse selection effects.

To conclude:

1. whenever the weight assigned to households's utilities in the SWF is more or less the same, the best monetary policy to implement is always a policy which ensures full revelation at equilibrium;

2. whenever more importance is attached to households $h = 0$ the best policy is a fully revealing monetary policy;

3. when households $h = 1$ are assigned the highest weight, the best policy is a fully non revealing monetary policy; also partially non revealing monetary policies which prevent the disclosure of individual $h = 2's$ signals improve their welfare;

Table 3: Values of the Social Welfare Function and Information

| Favourite Individual | Weights $\alpha^1$ | $\alpha^2$ | $\alpha^3$ | Value of the SWF | Degree of revelation of REE |
|---|---|---|---|---|---|
| h=0 best off | 0.999 | 0.005 | 0.005 | 3.59637 | fully revealing |
| h=0 worst off | 0.001 | 0.499 | 0.499 | 3.51955 | fully non revealing |
| h=1 best off | 0.004 | 0.992 | 0.004 | 4.22994 | fully non revealing |
| h=1 worst off | 0.499 | 0.002 | 0.499 | 3.13971 | fully revealing |
| h=2 best off | 0.004 | 0.004 | 0.992 | 2.81745 | fully non revealing |
| h=2 worst off | 0.499 | 0.499 | 0.002 | 3.90593 | fully revealing |
| equality | 0.333 | 0.333 | 0.333 | 3.49799 | fully revealing |
| h=2 worst off | 0.210 | 0.740 | 0.050 | 3.94545 | partially rev. $p_0(1) = p_0(3)$ |
| h=2 worst off | 0.130 | 0.800 | 0.070 | 4.03385 | partially rev. $p_0(2) = p_0(4)$ |
| h=2 worst off | 0.120 | 0.840 | 0.040 | 4.08000 | partially rev. $p_0(1) = p_0(2)$ |
| h=1 best off | 0.100 | 0.800 | 0.100 | 4.00100 | partially rev. $p_0(3) = p_0(4)$ |

4. whenever the emphasis in put on households $h = 2$ the best monetary policy is a fully non revealing.

# 10 Conclusions

We provide a general characterization of the structure of rational expectations equilibria for economies with an incomplete financial market. Specifically, we consider two-period, pure exchange economies with finitely many states of private information. Assets are nominal. The approach is minimal, under the assumptions of Polemarchakis and Siconolfi [21] needed to guarantee non revelation, we show that without loss of generality one single nominal asset suffices to characterize the generic dimension of the set of equilibrium allocations for any degree of information disclosure. While fully revealing rational expectations equilibria are finite and zero dimensional, fully non informative and partially revealing equilibria contain smooth submanifolds of strictly positive dimensions. In nominal asset economies with an incomplete asset market, prices and allocations are essentially determined by monetary policy. The fact that fully revealing rational expectations equilibria are finite and zero dimensional implies that under full private information disclosure competitive rational expectations equilibria are typically locally unique and monetary policy is neutral. However, the presence of (real) indeterminacy in noninformative economies calls for an explicit modelling of how money affects equilibrium outcomes.
Then, by means of different algorithms we show how a central bank deciding on the money supply, may affect the revelation of information in the economy. While ree are typically fully revealing, by specific indeterminacy parametrizations, that is by specific sequences of money supply, the central bank may lead the economy to non revealing ree.
Finally, by means of an example and numerical simulations, we show how the policy the central bank decides to implement, by determining the degree of pri-

vate information disclosure, also affects in different measure the welfare of the heterogeneous agents in the economy.

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