EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 830 / NOVEMBER 2007

THE TERM STRUCTURE OF EURO AREA BREAK-EVEN INFLATION RATES

THE IMPACT OF SEASONALITY

by Jacob Ejsing, Juan Angel García and Thomas Werner

EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 830 / NOVEMBER 2007

THE TERM STRUCTURE OF EURO AREA BREAK-EVEN INFLATION RATES

THE IMPACT OF SEASONALITY[1]

by Jacob Ejsing,
Juan Angel García
and Thomas Werner[2]

In 2007 all ECB publications feature a motif taken from the €20 banknote.

1 We are indebted to Refet Gürkaynak and Ivan Petrella for very useful discussions in the early stages of this project. We also thank an anonymous referee for helpful comments. Any remaining errors are our responsibility. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the European Central Bank.

2 All authors: Capital markets and financial structure division, DG-E, European Central Bank, Kaiserstrasse 29, 60311 Frankfurt am Main, Germany; e-mail: Jacob.Ejsing@ecb.int, Juan_Angel.Garcia@ecb.int, thomas.werner@ecb.int. Corresponding author: Thomas Werner.

Address
Kaiserstrasse 29
60311 Frankfurt am Main, Germany

Postal address
Postfach 16 03 19
60066 Frankfurt am Main, Germany

Telephone
+49 69 1344 0

Website
http://www.ecb.europa.eu

Fax
+49 69 1344 6000

Telex
411 144 ecb d

ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

CONTENTS

Abstract

This paper provides a toolkit for extracting accurate information about inflation expectations using inflation-linked bonds. First, we show how to estimate term structures of zero-coupon real rates and break-even inflation rates (BEIRs) in the euro area. This improves the analysis of developments in inflation expectations by providing constant maturity measures. Second, we show that seasonality in consumer prices introduces misleading and quantitatively important time-varying distortions in the calculated BEIRs. We explain how to correct for this in the estimation of the term structure, and thus provide a unified framework for extracting constant maturity BEIRs corrected for seasonality.

Keywords: *Term structure, break-even inflation rates, inflation-linked bonds, inflation seasonality*

JEL Classification: *E31, E43, G12*

Non-technical summary

Inflation-linked instruments have become an established asset class in major financial markets. From a monetary policy perspective, this is a welcome development, as the emergence of a well-developed inflation-linked bond market offers new possibilities for disentangling the information embodied in nominal bond yields. First, yields on inflation-linked bonds are interesting in their own right as they are interpretable as (ex-ante) real interest rates, providing valuable information about the state of and the market's outlook for the real economy. Second, they facilitate the calculation of indicators of inflation expectations. Indeed, the so-called break-even inflation rates, which reflect the yield spread between inflation-linked and comparable nominal bonds, have by now become standard indicators of inflation expectations regularly referred to by practitioners, media and policymakers.

This paper provides a toolkit to extract more accurate information about inflation expectations using inflation-linked bonds. We first show that, to fully exploit the information content of the prices of inflation-linked bond yields, it is necessary to take into account some key characteristics of those instruments. This involves a number of preliminary calculations and adjustments. For example, calculations based on specific inflation-linked bonds have the problem of the decreasing maturity of the real interest rates and break-even inflation rates based on them, which distorts their interpretation over long periods of time. In addition, the presence of seasonal factors in the underlying price index complicates further the measurement of inflation expectations implied by those instruments.

We show that the impact of inflation seasonality on real yields and break-even inflation rates are large enough to distort significantly the information content of these measures, in particular at short-to-medium maturities. We also document, as a cross-check of our approach, that the comovement between break-even inflation rates extracted from inflation-linked bonds and inflation swap rates becomes much stronger when the former rates are adjusted for seasonality. This is in line with intuition as inflation swap rates are, at least in principle, unaffected by seasonality because they refer to full-year maturities.

The proposed methodology for correcting the term structure of real yields and break-even inflation rates for those effects should be relevant for anyone interested in the information provided by inflation-linked bonds, from policymakers to market participants actively involved in the trading of those instruments.

1 Introduction

Inflation-linked instruments have become an established asset class in major financial markets. From a monetary policy perspective, this is a welcome development, as the emergence of a well-developed inflation-linked bond market offers new possibilities for disentangling the information embodied in nominal bond yields. First, yields on inflation-linked bonds are interesting in their own right as they are interpretable as (ex-ante) real interest rates, providing valuable information about the state of and market's outlook for the real economy. Second, they facilitate the calculation of indicators of inflation expectations (and associated risk premia). Indeed, the so-called break-even inflation rates (BEIRs henceforth) reflecting the yield spread between inflation-linked bonds and comparable nominal bonds have by now become a standard indicator of inflation expectations, regularly referred to by practitioners, media and policymakers. Comprehensive descriptions of the potential use of U.S. inflation-linked treasuries (TIPS) for extracting inflation expectations are provided by Sack (2000) and Sack (2002).

We make two main contributions to improve the calculation of inflation expectations extracted from bond market instruments. First, we explain how to estimate term structures of zero-coupon real and BEIRs in the euro area, which improves the analysis of developments in inflation expectations by providing constant maturity measures. In this respect we apply standard yield curve fitting methods as described by Gurkaynak, Sack, and Wright (2007) to inflation-linked bonds. Second, we show that inflation seasonality introduces potentially misleading and quantitatively important time-varying distortions in the calculation of BEIRs and explain how to correct for this in the estimation of the term structure. Those two contributions are relevant for anyone interested in the information provided by those instruments, from policymakers to market participants actively involved in the trading of those instruments. Furthermore, zero-coupon real rates are an indispensable input for term structure models disentangling real and nominal movements in the yield curve as in D'Amico, Kim, and Wei (2007).[1]

Our framework for extracting constant maturity BEIRs corrected for inflation seasonality improves over existing approaches in a number of ways. Evans (1998) for example proposes a method for estimating term structures of break-even inflation rates but abstracts from the impact of seasonality. In contrast, Gapen (2003) and Canty (2007) do present methods for dealing with inflation seasonality when computing BEIRs, but they focus on individual bonds and do not estimate term structures of BEIRs. This paper is therefore the first to propose a methodology which allows for tackling the problems of inflation seasonality and varying maturity of the bonds simultaneously.[2]

The paper is organized as follows. Section 2 presents a brief description of the market and characteristics of inflation-linked bonds in the euro area. The computation of term structures of real and BEIRs is described in Section 3. A framework for dealing with seasonality is presented in Section 4. In particular, an empirical evaluation of the magnitude of the distortions arising from inflation seasonality is also provided. Section 5 concludes.

2 The euro area inflation-linked bond market

Inflation-linked bond markets have experienced a significant growth in recent years all over the world. This is noteworthy for two reasons. First, the issuance of inflation-linked bonds appears to have gained momentum in a period in which central bank credibility and price stability are very high by historical standards. In this context, the issuance

[1]D'Amico, Kim, and Wei (2007) argue that correcting for the indexation lag (carry effect) is more important than correcting for seasonality which they treat as measurement errors. Contrary to this view we focuss on the seasonal adjustment because we found that for the euro area seasonal effects are very regular and not well described as measurement errors.

[2]Given the relative quantitative importance of its impact on the calculation of BEIRs and the complexities involved, we focus the paper on the estimation of term structures corrected for inflation seasonality. However we also provide a methodology to tackle the presence of the indexation lag in the appendix.

of these instruments has been motivated as a means to complete financial markets by providing an effective hedge against inflation in the long-term (especially in the context of pension fund management). Canada in 1991, the United States[3] in 1997 and more recently France in 1998, Greece and Italy in 2003, Japan in 2004 and Germany in 2006 started to issue inflation-linked bonds, while some other countries continued (U.K.) or revived (Australia) their issuance programmes with similar arguments. Second, inflation-linked bonds (ILBs henceforth) are much less novel than they are often believed to be.[4] The perception that inflation-linked bonds are a recent innovation owes to a large extent to the fact that they have not been extensively used in the history of finance.

Given the recent growth of this market, mainly fueled by institutional demand in the wake of new regulation, it has become worthwhile to develop a framework that allows for a thorough analysis of the information content of inflation-linked bonds. Although our proposed framework takes into account the specific characteristics of the euro area ILB market, the methodological contributions are readily applicable to other ILB markets.

The first bond whose coupon payments were indexed to euro area inflation was issued by the French Treasury in October 2001, with maturity July 2012 (OATei 2012). Following a relatively slow start, the market for inflation-linked bonds in the euro area has experienced a significant growth since 2003, and has become the second largest sovereign ILB market both in terms of outstanding volumes and turnover, only behind the US market. The ECB's quantitative definition of price stability refers to all-items HICP, but compliance with French regulations on the issuance of inflation-linked instruments has led to the choice of the euro area HICP index (ex. tobacco) as reference

[3] A comprehensive description of the U.S. market for inflation-indexed treasuries can be found in Sack and Elsasser (2004).

[4] A bond whose principal and interests were linked to the price of a basket of goods was already issued by the State of Massachusetts in 1780, and, in essence, the formulation of that contract captured all the essential features of inflation-linked bonds as they exist today.

Issuer	Maturity date	Issuance date	Amount outstanding (EUR billions)	Rating (S & P)	Annual coupon payment	Coupon frequency
Italy	15 Sep 2008	Sep. 2003	13.40	A+	1.65	semi-ann.
France	25 Jul 2010	Apr. 2006	6.50	AAA	1.25	annual
Italy	15 Sep 2010	Sep. 2004	14.30	A+	0.95	semi-ann.
France	25 Jul 2012	Nov. 2001	14.50	AAA	3.00	annual
Italy	15 Sep 2014	Feb. 2004	14.50	A+	2.15	semi-ann.
France	25 Jul 2015	Nov. 2004	10.40	AAA	1.60	annual
Germany	15 Apr 2016	Mar. 2006	11.00	AAA	1.50	annual
Italy	15 Sep 2017	Oct. 2006	10.05	A+	2.10	semi-ann.
France	25 Jul 2020	Jan. 2004	11.05	AAA	2.25	annual
Greece	25 Jul 2025	Mar. 2003	7.20	A (FIT)	2.90	annual
Greece	25 Jul 2030	Apr. 2007	3.50	A (FIT)	2.90	annual
France	25 Jul 2032	Oct. 2002	8.75	AAA	3.15	annual
Italy	15 Sep 2035	Oct. 2004	10.80	A+	2.35	semi-ann.
France	25 Jul 2040	Mar. 2007	4.00	AAA	1.80	annual

Table 1: Sovereign bonds linked to the euro area HICP excluding tobacco. Source: Reuters, August 2007.

index.[5] This index has become the market benchmark in the euro area since then, and all the inflation-linked government bonds indexed to euro area inflation issued so far use this as reference index. It has also become the standard reference index for other financial products, most notably inflation-linked swaps.

Table 1 summarises some of the key characteristics of the existing inflation-linked bonds in the euro area. In addition to France, three additional euro area countries, namely Greece, Italy and Germany, have issued inflation-linked bonds, and a few other

[5] The issuance of sovereign bonds linked to euro area inflation followed the introduction of bonds indexed to the French CPI excluding tobacco (OATi's) in 1998. It was clear at the time that the ECB's definition of price stability in the euro area would be based on the Harmonised Index of Consumer Prices (HICP), an index regularly published by Eurostat, but the choice of the French CPI as reference index was largely motivated by the lack of a track record of the HICP prior to 1999.

euro area governments have expressed interest in the issuance of inflation-linked debt. Liquidity in the euro area inflation-linked bond market has been enhanced by the higher number of issuers and maturities available, and turnover has increased substantially in the last few years, see Figure 1.[6] Moreover, the outlook for the euro area inflation-linked market is still promising, as demand is likely to remain strong in the future.[7]

To develop a framework that allows for a thorough analysis of the information content of the euro area ILB market, it is important to bear in mind some of the main characteristics of ILBs in general and euro area ILBs in particular, for they are crucial to understand the methodology we propose in the next sections. We analyse them in turn.

First, as highlighted in Table 1 above, the euro area ILB market comprises several different issuers. The Italian, Greek and German bonds share most of the technical characteristics of the existing French inflation-linked bonds (to be described in detail below), namely they are linked to the euro area HICP (ex. tobacco) and also offer guaranteed redemption at par, implying deflation protection. However, the Italian and Greek bonds are perceived by rating agencies to involve different credit risk than the French and German bonds (see Table 1 for specific details). In addition, reflecting country-specific standard practices, coupon payments for the Italian inflation-linked bonds take place at semi-annual frequency, rather than at the annual frequency of the French, Greek and German bonds. Note also that, although the Italian and French trea-

[6]Some changes in regulations seem to have played a major role in boosting demand for such instruments, mainly from insurance companies and pension funds, and may have led to some shortages in the market despite the growing issuance volume (see García and van Rixtel (2007), for further details).

[7]Although French institutional investors, particularly insurers, banks and mutual funds, have been the main investors in continental Europe, pension funds from other continental European countries are likely to become more active in the future, in response to potential pension reforms and the related need to hedge long-term inflation-indexed liabilities. Available evidence from the recent launch of a euro area long-term inflation-linked bond, the OATei 1.80% 25 July 2040, supports this view: a large proportion of the order book (25%) was allocated to asset managers, pension funds and insurance companies.

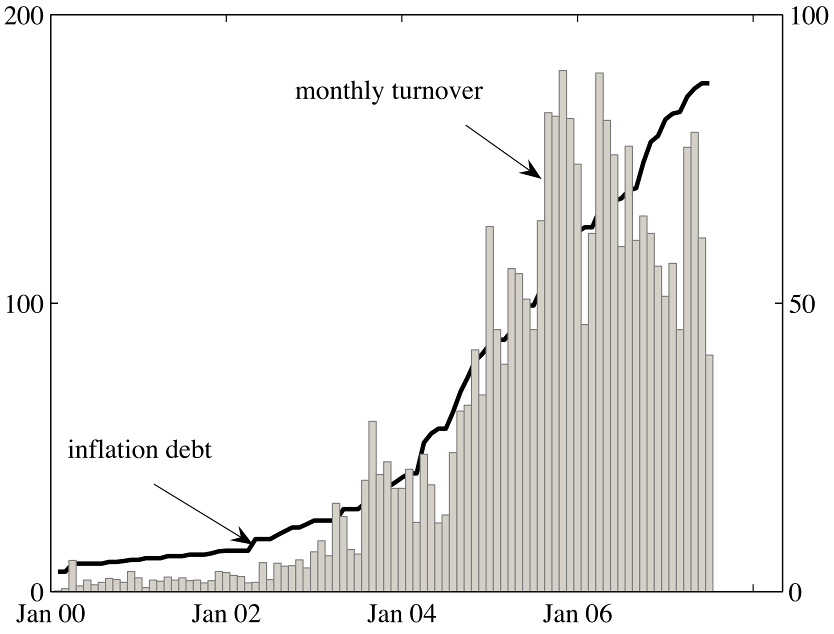

Figure 1: Amount outstanding (LHS-scale) and monthly turnover of sovereign French bonds indexed to the euro area HICP (3-month moving average, RHS-scale)

suries have taken significant steps towards "completing a curve" with their respective instruments over the last few years, the distribution of available maturities in the euro area ILB market has been strongly related to issuers, and therefore bond characteristics, until very recently. The heterogeneity of euro area ILBs can introduce significant distortions in the estimation of the real and BEIR curves, but existence since 2006 of six AAA-bonds, namely the French bonds maturing in 2010, 2012, 2015, 2020 and 2032 and the German bond maturing in 2016, offers the attractive possibility of estimating a pure AAA-curve.[8]

Second, euro area ILBs share the characteristics of the so-called Canadian model by which there is a "three-month" indexation lag[9] in the implied contract to account for

[8]Note that by focusing on AAA-instruments not only rating differences but also coupon frequency heterogeneity are avoided.

[9]The indexation lag implies that yields on ILBs - and thus BEIRs derived from them - are not fully forward-looking. In fact, break-even inflation rates based on them will also contain inflation realised in the three months preceding the pricing date.

[10]See Appendix for further details and a numerical example.

the delays in the official releases of the price indices by statistical agencies (in the case of the euro area releases take place around the 15th of the month and correspond to the price index of the previous month). Releases of HICP (ex. tobacco) index levels (I) refer to monthly figures, so daily price index values are calculated by linear interpolation between the last two available released index values at a point in time. Taking into account publication lags, this results in a maximum lag for the reference index of three months. Formally, on the first day of a given month (M) the corresponding (daily) price index P_t^{lag} is the (published) price index value three months ago (I_{M_t-3}). As the month advances the daily price index values are computed by linear interpolation, as a weighted mean of the published price indexes of the third and second previous month. Formally,[10]

$$P_t^{lag} = I_{M_t-3} + \frac{No.\ days\ since\ start\ of\ current\ month\ -1}{No.\ days\ of\ current\ month}\left[I_{M_t-2} - I_{M_t-3}\right].$$

Third, the (dirty) price of an ILB comprises both a (real) interest rate accrual and an inflation accrual corresponding to a compensation for realised inflation. Formally, the dirty price of an ILB is given by

$$[B_t(H) + AI_t]\,\frac{P_t^{Lag}}{P_0} \tag{1}$$

where accrued interest, AI, is calculated in the standard way

$$AI_t = \left[\frac{No.\ days\ elapsed\ in\ coupon\ period}{No.\ days\ in\ coupon\ period}\right] \cdot C$$

with C being the bond coupon rate. The inflation accrual is obtained through multiplication with the term $\frac{P_t^{Lag}}{P_0}$, which is always known at the settlement date because of the indexation lag.

Finally, government bonds linked to euro area inflation all refer to the seasonally unadjusted price level, which implies that extracted real rates and BEIRs will be affected by that seasonality, unless appropriate adjustments are made.

In the following two sections we show how the challenges posed by these characteristics of (euro area) ILBs can be handled in a consistent framework, which allows for the estimation of term structures of real rates and BEIRs.

3 Estimating term structures of real and break-even inflation rates

In this section we discuss the computation of constant-maturity zero-coupon real rates and BEIRs. For the sake of clarity, in this section we will abstract from other potential distortions in the estimation, like inflation seasonality and the indexation lag. The issue of seasonality will be extensively addressed in the next section.

3.1 The computation of real and break-even inflation rates

We first briefly summarise the conventional way of calculating yields to maturity for inflation-linked bonds. Prices of inflation-linked bonds ($B_{t,i}$) are usually quoted as a fraction of the face value excluding indexation.[11] This means that quoted prices are "clean" not only in the sense that they do not include interest accrual, but also clean in the sense that they are free of inflation compensation. The quoted prices of inflation-linked bonds can therefore be considered as "double clean" prices, while the price investors actually have to pay reflects both the interest accrual ($AI_{t,i}$) and the inflation accrual (P_t^{Lag}/P_0), where P_0 denotes the base price index for the indexation. The payable (nominal) price of an inflation-linked bond is therefore $(B_{t,i} + AI_{t,i})(P_t^{Lag}/P_0)$. At coupon dates the holder of the bond gets the coupon C_i times the inflation accrual

[11] The actual quotation is in percentage of the face value of 100. To simplify the notation in the following we assume that the face value is one and prices are quoted as a fraction of one.

P_{t+h}^{Lag}/P_0. The same applies to the redemption date[12]. We define $Q_t^{IL}(h^*)$ to be the nominal price at time t of a "hypothetical" zero-coupon inflation-linked bond paying P_{t+h}^{Lag}/P_t^{Lag} euro in h days, i.e. at time $t+h$. We use h^* as short-hand notation for $h^*(h) = h/365.25$ and define the time to maturity measured in years analogously as $H^*(H) = H/365.25$.[13] Therefore, an inflation-linked bond can be valued by

$$\left[B_{t,i} + AI_{t,i}\right] \frac{P_t^{Lag}}{P_0} = C_i \sum_{h=1}^{H_i} I_{t,i}(h) \frac{P_t^{Lag}}{P_0} Q_t^{IL}(h^*) + \frac{P_t^{Lag}}{P_0} Q_t^{IL}(H_i^*). \tag{2}$$

where H_i is the time to maturity of bond i measured in days, and $I_{t,i}(h)$ is an indicator variable which takes the value one if a coupon is paid at time h and zero otherwise. The terms P_t^{Lag}/P_0 on the right hand side of equation (2) appear because the actual inflation accrual a bond holder gets at time $t+h$ is P_{t+h}^{Lag}/P_0 instead of P_{t+h}^{Lag}/P_t^{Lag} as implied in our definition of $Q_t^{IL}(h^*)$. However, our definition of the discount factor $Q_t^{IL}(h^*)$ of an inflation-linked bond is convenient because using the quoted (double clean) prices, an inflation-linked bond can be valued like an ordinary nominal bond. Indeed, equation (2) can be rewritten as

$$B_{t,i} + AI_{t,i} = C_i \sum_{h=1}^{H_i} I_{t,i}(h) Q_t^{IL}(h^*) + Q_t^{IL}(H_i^*). \tag{3}$$

The corresponding discount factor $Q_t^{IL}(h^*)$ can be interpreted as a real discount factor[14]. The yield-to-maturity, R^{IL}, can be straightforwardly computed by numerically solving equation (3) assuming a constant discount rate.

[12]Note also that ILBs typically are equipped with a "deflation floor" on the principal, in the sense that redemption at par is guaranteed. As this option element has negligible value in the euro area, as it would require average deflation over the entire life of the bond, we abstract from this in the calculations. Although not relevant in our sample period, it should be noted that during periods of pronounced "deflation scare", the prices of recently issued (short-maturity) bonds may be somewhat affected by the deflation floor.

[13]So, where h is a number of days, h^* is simply the same period measured in years. This notation is introduced to make standard expressions of annualised yields compatible with sums over cash-flows using number of days as the discrete counting variable.

[14]As mentioned above, the indexation lag implies that the derived discount factors are not real discount factors in the strict sense, as they also contain three months of past, yet still partially unknown, inflation. Adjusting for this requires a model to forecast "real-time" inflation. In the absence of reliable market-based expectations of the current price level, it does not appear worthwhile to introduce an additional layer of model-dependence in the extraction of BEIRs. Given that the quantitative impact of seasonality is likely to be substantially larger, our focus is on correcting for seasonality. See the appendix for a formal statement of the relation between different notions of "real" rates.

The most common way to compute BEIRs has been to simply subtract the yield-to-maturity on a specific inflation-linked bond, from the yield-to-maturity R on a specific nominal bond with comparable maturity (BEIR $= R - R^{IL}$). However, the estimation of a term structure of zero-coupon real rates and corresponding BEIRs offers two major advantages. First, it allows the calculation of time series of real yields and BEIRs with constant maturity, which is particularly useful when assessing developments over a relatively long period of time. The maturity of observed yields and rates from existing bonds, by contrast, is not constant but declines over the existence of the bonds, which may complicate the interpretation of yield developments. Second, the calculation of zero-coupon rates allows potential distortions related to the different durations of the bonds used in the calculation of BEIRs to be avoided. Such distortions are related to the different cash-flow structures of inflation-linked and nominal bonds.[15]

3.2 Estimating term structures of real yields and inflation

Constant-maturity zero-coupon BEIRs can be constructed by subtracting zero-coupon real rates from zero-coupon nominal rates of the same maturity. Hence, the problem of computing constant-maturity zero-coupon break-even rates reduces to estimating real and comparable nominal zero-coupon yield curves. The literature on yield-curve estimation proposes a variety of methods which can be roughly divided into parametric and non-parametric methods. In the case of parametric approaches, parsimoniously parameterised functional forms of the yield curve are assumed and the parameters of these functions are chosen by optimising the fit to the observed bond prices. Usually

[15] (Macaulay) duration is defined as the weighted average maturity of a bond's cash-flows, where the weights are the present values of each of the payments as a proportion of the total present value of all cash flows.

only a small number of parameters has to be estimated. On the other hand, non-parametric approaches are more flexible in fitting observed bond prices. More flexibility, however, may come at the price of potential over-fitting (i.e. effects on the estimated yield curve of potential errors in bond prices that should be smoothed out), which is an especially important problem for the estimation of the real yield curve due to the rather few inflation-linked bonds available.

In the light of those considerations and the limited number of available bonds, we have opted for estimating the nominal yield and real curves by a fairly standard parametric approach first introduced by Nelson and Siegel (1987).[16] Hence, we specify the following functional form for the h^*-year zero-coupon yield on inflation-linked bonds $y_t^{IL}(h^*)$:

$$y_t^{IL}(h^*) = \beta_1 + (\beta_2 + \beta_3)\frac{\tau}{h^*}\left(1 - \exp\left(-\frac{h^*}{\tau}\right)\right) - \beta_3 \exp\left(-\frac{h^*}{\tau}\right). \tag{4}$$

Using the relation between bond prices and bond yields ($Q^{IL}(h^*) = e^{-h^* y_t^{IL}(h^*)}$) equation (4) determines the price $Q^{IL}(h^*)$ of a zero-coupon real bond with h^* years to maturity. A real coupon bond with maturity H_i^* years (or H_i days) can be considered as a portfolio of zero-coupon real bonds and valued accordingly:

$$B_{t,i} + AI_{t,i} = \sum_{h=1}^{H_i} I_{t,i}(h) Q_t^{IL}(h^*) C_i + Q_t^{IL}(H_i^*) + \epsilon_{t,i}. \tag{5}$$

The left-hand side of equation (5) is the (dirty) price of a real coupon bond which should equal the price of a portfolio containing a sequence of coupon payments C_i and the principal payed at maturity H_i^*. The theoretical value of the coupon bond is thus a function of the parameters $\beta_1, \beta_2, \beta_3$, and τ in equation (4). Therefore, given a certain number of bonds, the parameters of the real yield curve can be easily estimated by minimizing the (weighted) sum of squared estimation errors ϵ_i.

Using the estimated parameters of equation (4) it is possible to compute constant-maturity zero-coupon real interest rates and, of course, the whole real yield curve.

[16]The Nelson and Siegel method is commonly used by Central Banks for estimating nominal term structures, see BIS (2005).

Furthermore, as mentioned above, corresponding constant-maturity zero-coupon BEIRs can be computed using a similarly extracted nominal yield curve.

3.3 Data and estimation results

The number of available bonds linked to euro area HICP is still rather limited (see section 2 for details), which complicates somewhat the estimation of the term structures. One possibility would be to use all available euro area government bonds linked to euro area HICP, but, as shown in Table 1, this would however result in using bonds with different credit ratings. Alternatively, focusing only on AAA-rated bonds comes at the cost of reducing substantially the number of available bonds. Practical considerations led us to use a mixed strategy. Most of the empirical results presented in this paper refer to the period starting on 16 May 2006 since which the curve is only based on AAA-rated bonds. Before that date however, we found that it was preferable to include the Italian 2008 ILB to effectively pin down the short-end of the curve and obtain reliable estimates of the yield curve model parameters. Prices on ILBs and comparable nominals are mid-quotes provided by Reuters.

The estimation of term structures allows us to obtain new insights on the changes in real yields and BEIRs over recent years. For example, Figure 2 shows the estimated constant-maturity zero-coupon real rates for five and nine year maturities together with the yield on the inflation-linked French bond maturing in 2012. The figure illustrates the potential insights offered by constant-maturity interest rates: in 2004 the remaining maturity of the French 2012 bond was between 8 and 9 years, and indeed the yield on this bond was close to the estimated nine-year maturity zero-coupon real interest rate. Later on, in parallel with the decrease in the remaining maturity, the yield on the French 2012 bond approached the five-year constant-maturity zero-coupon real interest rate, which illustrates that the evolution of the observed inflation-linked bond yields at least partially reflects the gradual contraction of the maturity over time. The computation of constant-maturity real interest rates allows for disentangling this effect from “fundamental” movements in the yields. Figure 2 also shows that the magnitude of the maturity effect depends on the slope of the yield curve. In 2004 when the real yield curve was rather steep (see also Figure 3), the maturity effect appeared to be stronger than towards 2006, when the real yield curve was essentially flat. Second,

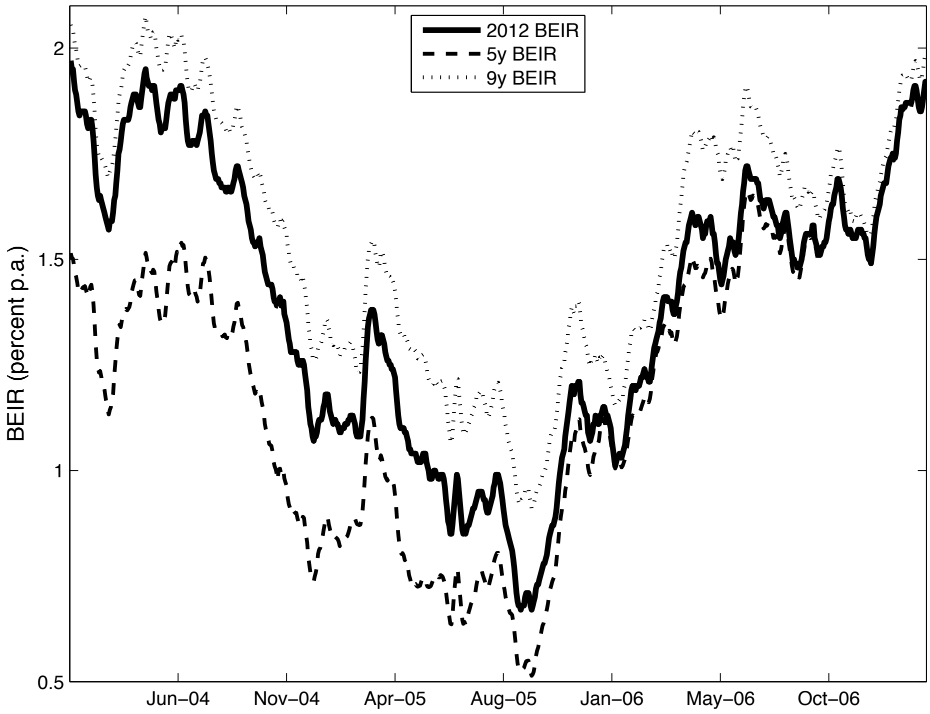

Figure 2: Yield on the French 2012 inflation-linked bond (percent p.a.)

our estimates also allow for an analysis of the information incorporated in inflation-linked bonds beyond yields on single bonds. Figure 3, for example, illustrates the above mentioned flattening of the real yield curve observed over recent years. In addition, the term structure of BEIRs, computed as the difference of the nominal yield curve and the real yield curve opens new possibilities to analyse movements in inflation expectations (and related premia) among market participants.[17] Figure 4 compares the estimated

[17]We estimate the nominal yield curve using nominal bonds with similar maturities to those of the bonds used in the estimation of the real curve to avoid distortions in the BEIR curve induced by different weights on fitting the various maturities for the real and nominal yield curves. Given the larger selection of nominal bonds, we use more bonds for the estimation of the nominal than the index-linked curve.

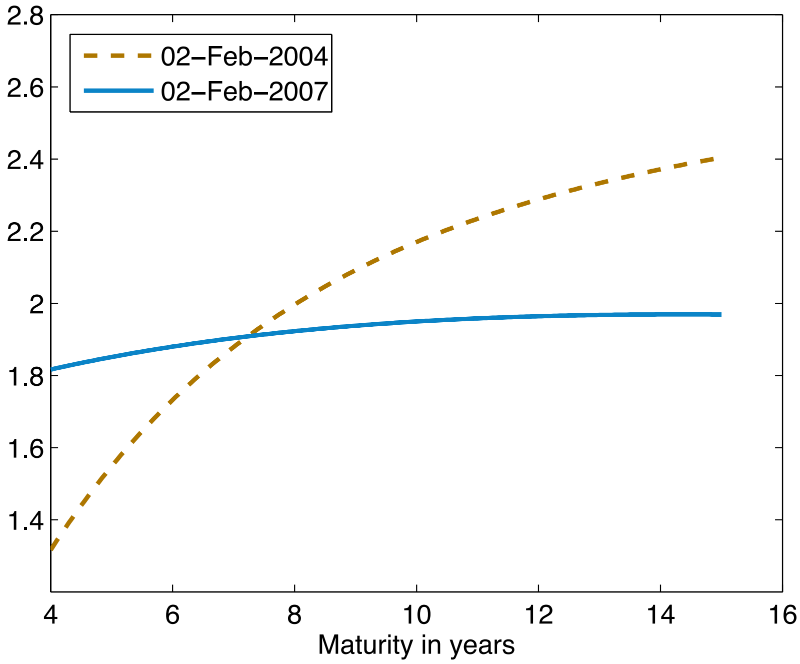

Figure 3: Real yield curves for 2004 and 2007 (percent p.a.)

break-even curve at the beginning of 2004 with the corresponding curve at the beginning of 2007. Similar to the real yield curve, the break-even inflation curve has flattened considerably over the recent years.

4 Correcting for inflation seasonality

Inflation exhibits seasonal fluctuations, and those seasonal fluctuations have an impact on the prices of inflation-linked bonds because their cash-flows are linked to the seasonally *unadjusted* price level. Therefore, accounting for the seasonality in consumer prices is an important issue both for the correct valuation of inflation-linked bonds and for the extraction of BEIRs. To avoid misinterpreting changes in BEIRs caused by predictable seasonal variations in consumer prices as changes in the underlying growth

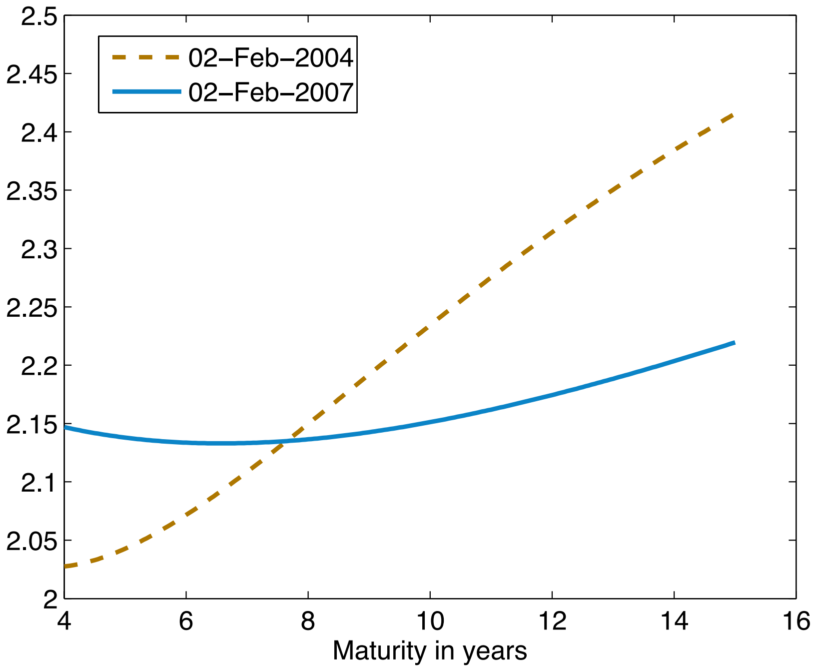

Figure 4: Break-even inflation curves for 2004 and 2007 (percent p.a.)

trend of prices, we here advocate the seasonal adjustment of the prices of inflation-linked bonds before estimating the term structure of zero-coupon real yields. In addition, we also provide some quantitative evidence of the impact of the distortions generated by inflation seasonality in the calculation of BEIRs.

4.1 Inflation seasonality and the yields of inflation-linked bonds: the case for correction

Overall consumer price indices like the HICP generally contain non-negligible seasonal patterns caused by factors such as periodic sales and strong seasonal fluctuations in the prices of certain components, such as food prices. In the euro area, the January price level, for instance, typically lies noticeably below the trend level of prices due to the inclusion of winter sale prices in its calculation. In contrast, index price levels in the second quarter of the year tend to lie above the general trend level of prices. As an illustration, Figure 5 depicts the seasonal component in euro area HICP (ex. tobacco), and shows that the amplitude of fluctuations has clearly increased in recent years.[18]

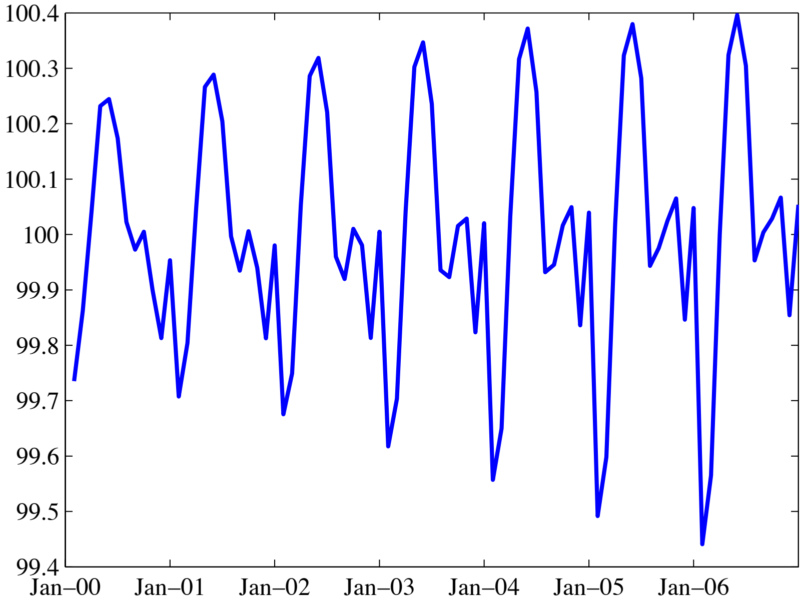

Figure 5: Multiplicative seasonal component of the euro area HICP (ex. tobacco) estimated with the X12-ARIMA methodology.

BEIRs calculated from bonds linked to the non-seasonally adjusted price index will naturally inherit that seasonality. To see this, assume first for simplicity that the (annual) coupon payments of the bonds used in the calculation take place on the same date of the year. Consider then the estimation of a zero-coupon BEIR curve at a given point in time within the year, say in January, from a series of ILBs whose coupon

[18] One likely reason for this is the wider coverage of periodic sales (e.g. in January) in the consumer price statistics across euro area countries, which has increased the observed seasonality. As the inclusion of sales in the computation of price indices has not been implemented at the same time in all euro area national statistical agencies, the increase in observed seasonality has taken place only gradually, depending on the seasonal factors of the sub-indices in each euro area country and the weight of the country in the overall index. For more technical information on estimating seasonal patterns in the euro area HICP see ECB (2000).

payments take place for example in July (as for the French Treasury bonds linked to euro area HICP). Given that the seasonal component of inflation tends to be higher in the first half of the year than in the second half, the inflation accrual also tends to be higher in the first half of the year, thereby affecting the prices of the bonds, and consequently the estimation of BEIRs. For example, if instead of January one estimates the BEIR curve in May, where the seasonal factor in euro area inflation tends to reach its peak, the estimated BEIRs would tend to be too low, from the perspective of seasonally-adjusted inflation expectations.

The impact of seasonality on the price of the bond clearly decreases with its maturity. In the example where BEIRs are calculated in January, the relative over-representation of the period of (relatively) high inflation accrual in the first half of the year will be lower the larger the number of (full-year) coupon payments decline with maturity. Figure 6 illustrates the effects of seasonality on BEIRs of different maturities. The figure is constructed by assuming a constant inflation rate of 2% and computing the yield effects implied by the seasonal pattern of euro area HICP (ex. tobacco) for 2006. It is striking how strongly the seasonality affects the BEIR curve, particularly at the short-end. The effect of seasonality on the seasonally unadjusted inflation-linked yield curve would be the same, but with the opposite sign.

Although official consumer price statistics are indeed seasonal and the inflation-linked bonds are indexed to the non-seasonally adjusted HICP (ex. tobacco), there are conceptual and technical reasons why correcting inflation-linked bond prices and BEIRs for inflation seasonality is advisable. As regards conceptual considerations, trading in those instruments obviously requires taking into account the effects that inflation seasonality has in the inflation accrual to correctly price the bond. Furthermore, from a medium-term monetary policy perspective, it is the trend growth of prices which is of main interest, rather than the seasonal fluctuations of inflation, and indeed, the inflation objective of the ECB (and other central banks) is stated in terms of the year-on-year growth rate of consumer prices, which by construction is unaffected by seasonality.

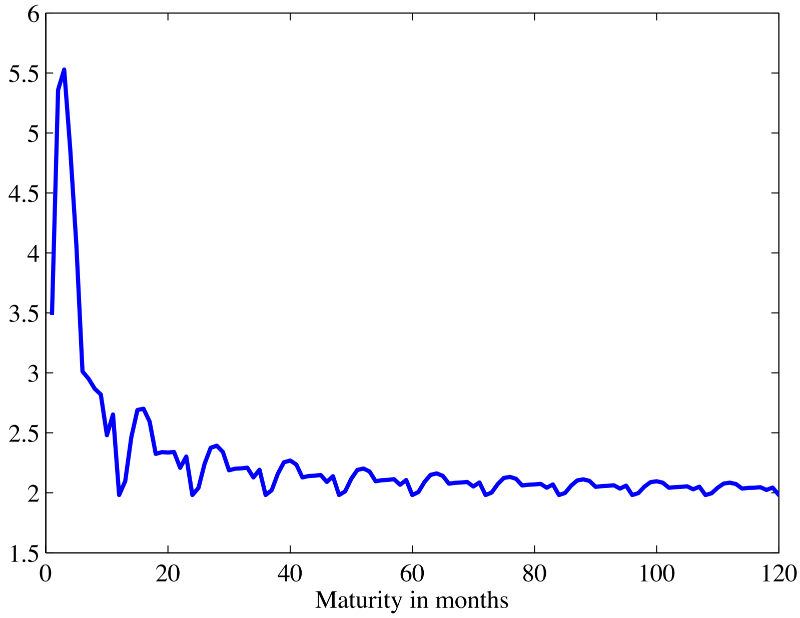

Figure 6: The effect of seasonality (in percent p.a.) on a theoretical break-even inflation curve assuming 2% annual inflation and using the 2006 vintage of seasonal factors for HICP (ex. tobacco)

From a technical perspective, the problem of fitting the inflation-linked yield curve without seasonal adjustment becomes increasingly problematic as some of the bonds come close to maturity: since the impact of seasonality is inversely related to the remaining time to maturity (see Figure 6), the short-end of the estimated inflation-linked yield curve will necessarily become increasingly distorted. If there were plenty of bonds of different maturities to choose from, the most problematic ones could be replaced but, given the still limited number of inflation-linked bonds in the euro area, this is likely to pose some serious problems in a not-so-distant future. Moreover, even with a large number of available bonds, standard procedures for yield curve fitting, such as the Nelson-Siegel methodology discussed in the previous section, will have difficulties in capturing the highly erratic shape of the unadjusted inflation-linked yield curve. Therefore, if no seasonal adjustment is applied, the curve one ends up estimating will depend on the specific mix of different seasonal factors in the portfolio of bonds underlying the yield curve estimation.

4.2 Seasonal adjustment of yield-to-maturity on individual bonds

Although the main focus of this paper is the estimating zero-coupon real and BEIR term structures, it is useful to consider first how yields-to-maturity on specific coupon bonds can be adjusted for seasonality. Most importantly, this will allow for a longer sample period than what is feasible with the full AAA zero-coupon curve (where estimation begins 16 May 2006). Fortunately, the approach for coupon bonds fully mirrors the procedure used for zero-coupon curve estimation, and just involves multiplication of the (dirty) market price by the ratio of seasonal factors before computing the yield-to-maturity.[19] Figure 7 shows the difference between adjusted and unadjusted BEIR on the French 3.0% July 2012 government bond as well as a projection of this difference into the future assuming the seasonally adjusted yield as well as seasonal factors remain constant over the remaining life of the bond. It is seen how the effect of seasonality has gradually increased, due to the joint effects of shrinking time-to-maturity and the increasingly pronounced seasonality in HICP. The projection shows that this effect will necessarily become dramatic as the bond comes even closer to maturity.

4.3 Estimating a seasonally-adjusted term structure

From a practical point of view, the estimation of a BEIR curve adjusted for inflation seasonality requires an ex-ante adjustment to bond prices. Recall that in Section 3, $Q_t^{IL}(h^*)$ was defined to be the nominal price at time t of a zero-coupon bond paying $P_{t+h}^{Lag}/\ P_t^{Lag}$ in h days. To obtain a seasonally-adjusted yield curve, we simply define the seasonally adjusted discount function, $Q_t^{IL,SA}(h^*)$, as the nominal price at period t of a zero-coupon bond paying $P_{t+h}^{Lag,SA}/\ P_t^{Lag,SA}$ in h days.

[19] See also Canty (2007) for a more thorough discussion of the computation of yields to maturity on inflation-linked coupon bonds.

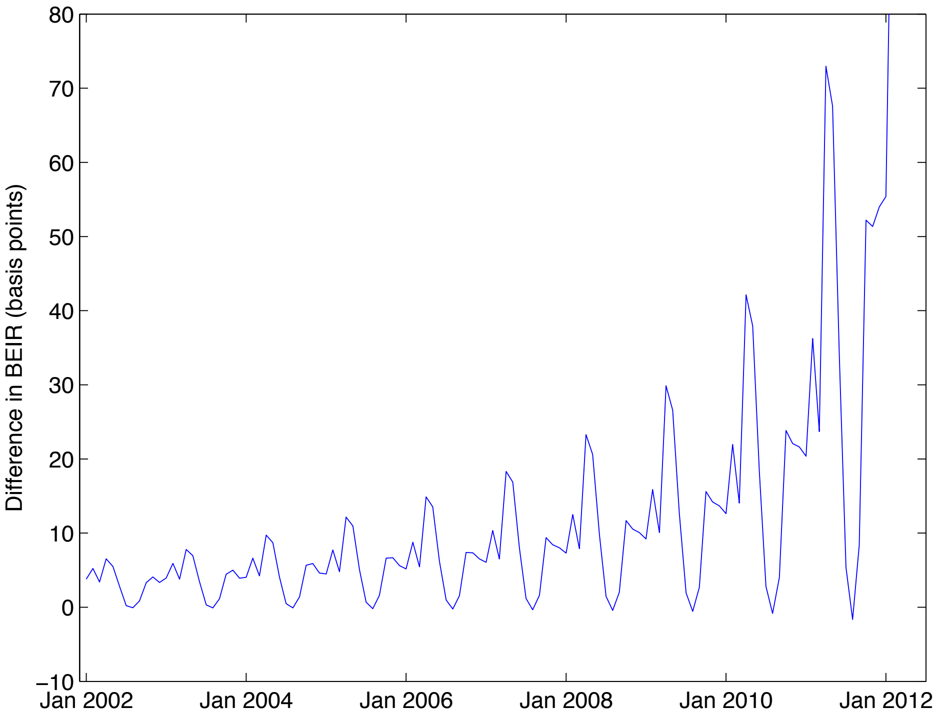

Figure 7: Difference between seasonal unadjusted and adjusted BEIRs (in basis points) for the OATei 3% 2012. A positive value implies that the unadjusted BEIR overstates inflation. The difference is based on estimated curves until mid-2007 and extrapolated thereafter assuming constant inflation.

We assume that seasonality takes a multiplicative form[20], such that seasonal factors link seasonally adjusted and unadjusted published price levels (I) as follows

$$I_t = I_t^{SA} SF_t \tag{6}$$

where I_t^{SA} denotes the seasonally adjusted price level. Applying the interpolation method for the lagged price indices (P^{Lag}) to their corresponding seasonal factors see equation 10 and the Appendix), we can write the payoff from the zero-coupon bond as

$$\frac{P_{t+h}^{Lag}}{P_t^{Lag}} = \frac{P_{t+h}^{Lag,SA}}{P_t^{Lag,SA}} \frac{SF_{t+h}^{Lag}}{SF_t^{Lag}} \tag{7}$$

[20]See Canty (2007) for a discussion of multiplicative versus additive seasonality in relation to BEIRs

Therefore, given an estimate of the ratio of the seasonal factors $SF_{t+h}^{Lag}/SF_t^{Lag}$, we can express the seasonally-adjusted inflation-linked discount function as follows:[21]

$$Q_t^{IL}(h^*) = \frac{SF_{t+h}^{Lag}}{SF_t^{Lag}} Q_t^{IL,SA}(h^*) \tag{8}$$

Substituting this into the relation used for fitting the yield curve gives

$$B_{t,i} + A_{t,i} = C_i \sum_{h=1}^{H_i} I_{t,i}(h) \frac{SF_{t+h}^{Lag}}{SF_t^{Lag}} Q_t^{IL,SA}(h^*) + \frac{SF_{t+H_i}^{Lag}}{SF_t^{Lag}} Q_t^{IL,SA}(H_i^*) + \epsilon_{t,i}. \tag{9}$$

Assuming that market participants expect seasonal factors to stay constant over the remaining life of the bond, i.e. $SF_{t+h}^{Lag} = SF_{t+H_i}^{Lag}$ for all yearly coupon dates h, the following simpler expression can be used for bonds with yearly coupon payments

$$[B_{t,i} + A_{t,i}] \frac{SF_t^{Lag}}{SF_{t+H_i}^{Lag}} = C_i \sum_{h=1}^{H_i} I_{t,i}(h) Q_t^{IL,SA}(h^*) + Q_t^{IL,SA}(H_i^*) + \epsilon_{t,i}. \tag{10}$$

The implementation of the above approach requires estimates of seasonal factors in euro area inflation. In our calculations we use estimates of the seasonal factors regularly computed by the ECB on the basis of X12-ARIMA methodology for seasonal adjustment.[22] Seasonally-adjusted real and BEIRs curves can then be estimated using equation (10). Unlike the standard seasonal adjustment of time series, the adjustment for inflation seasonality in BEIRs does not imply that the adjustments over a period of a year add up to zero. For example, in the case of French inflation-linked bonds, which mature (and pay coupons) on 25 July, the seasonal factor at maturity is the one corresponding to 25 April owing to the three-month indexation lag. Because the

[21] This rules out any (pricing of) risk related to changes in future seasonality.

[22] The daily series of seasonal factors used in this paper is based on 2006 seasonal factor estimates, applying the same interpolation methodology used for the level of the price index (see equation 10). Obviously, updates of the estimated seasonal factors can be incorporated with the release of new price information, but this is not necessary for our purpose here. The seasonaly-adjusted series of HICP (ex. tobacco) index is publicly available at the ECB webpage http://sdw.ecb.int/ under the code ICP.M.U2.s.X02200.3.INX. Seasonal factors can be easily calculated using the corresponding non-seasonaly-adjusted figures published by Eurostat.

April seasonal factor for the euro area HICP (ex. tobacco) is higher than the average seasonal factor, adjusted BEIRs based on the French bonds will, for most of the year, be lower than unadjusted rates (see Figures 6 and 7). Instead, for the German 2016 inflation-linked bond, which is indexed to the (seasonally low) January price level, the opposite is true. Since our BEIR curve uses information from both French and German bonds, but most bonds are form the French Treasury, the seasonally adjusted BEIRs tend to lie below the unadjusted BEIR curve.

We are now in a position to assess the quantitative implications of inflation seasonality on the estimated spot and implied forward BEIRs. Figure 8 (top panel) shows the differences in basis points between seasonally adjusted and unadjusted spot BEIRs for five and ten-year maturities as well as the five-year forward BEIR five years ahead. Several insights can be obtained from this evidence.

First, as suspected, there is a clear seasonal pattern in the differences between adjusted and unadjusted spot BEIRs. Specifically, unadjusted BEIRs tend to increase markedly relative to the adjusted figures during the spring, with the difference reaching a maximum at about 20 basis points for the five-year maturity in April. Besides the size of those differences, it is also important to note that they can distort significantly the interpretation of movements in BEIRs over certain periods of time: for instance, while unadjusted spot BEIR would suggest that market participants inflation expectations declined slightly in the Spring 2007 period, adjusted BEIR suggests that instead they steadily rose over the same period (see Figure 8, lower panel).

Second, spot BEIRs are significantly more affected by seasonality than BEIR forward rates. Indeed, in theory, forward BEIRs should be unaffected by inflation seasonality.[23] However, this theoretical result requires a highly accurate estimation of the *seasonally unadjusted* yield curve, which in turn would require a very large number of

[23]It can easily be shown that multiplying two zero-coupon bond prices by the same quantity does not affect the forward rate computed on the basis of these prices. This statement, however, refers only to forward rates computed from two zero-coupon BEIRs with a full number of years to maturity.

bonds.[24] In practice, as shown in the top panel of Figure 8, adjusted and unadjusted forward BEIRs are likely to differ slightly when using bonds with different seasonals for estimating the curve. In our specific case, this relates to the fact (explained above) that unadjusted BEIRs based on the French bonds tend to lie above their adjusted counterparts - and vice versa for the German bond. Since the 10-year segment of the AAA-curve in our estimation period was pinned down jointly by the French 2015 and German 2016 bonds this implies that the unadjusted 10-year BEIRs is lower than it would be in the case of a purely French curve. Since the 5-year segment was determined mainly by the French 2012 bond, this leads to a "low" unadjusted 5-year forward 5 years ahead BEIR. This explains why, with our specific selection of bonds, the unadjusted forward rate is slightly below the adjusted one, while the opposite is true for the spot rates. The effect of seasonality on the forward rate changes over time owing to, among other things, changes in HICP seasonal factors. It is important to note, however, that the differences between the adjusted and unadjusted BEIR curves (induced by the specific selection of bonds) reflect distortions in the unadjusted curve. The seasonally-adjusted BEIR curve should in principle not be affected by the bonds with different seasonals.

4.4 Comparison with inflation swaps rates

Exploiting the combined benefits of our constant-maturity BEIRs and the seasonal adjustment, we can now offer a comparison of the term structure of seasonally-adjusted BEIRs with a term structure of BEIRs extracted from zero-coupon inflation swaps. Since the inflation legs of the swaps contracts are linked to year-on-year increases in HICP, they should in principle be unaffected by the systematic intra-year variations in the price level. Moreover, by construction, quoted swap rates have constant maturity.

[24]The practical infeasibility of estimating a valid unadjusted curve is also one reason why adjusted BEIRs cannot be obtained simply by applying standard statistical procedures for seasonal adjustment to estimated yields ex-post. The seasonal adjustment must be done before fitting the curve, i.e. to prices rather than to yields.

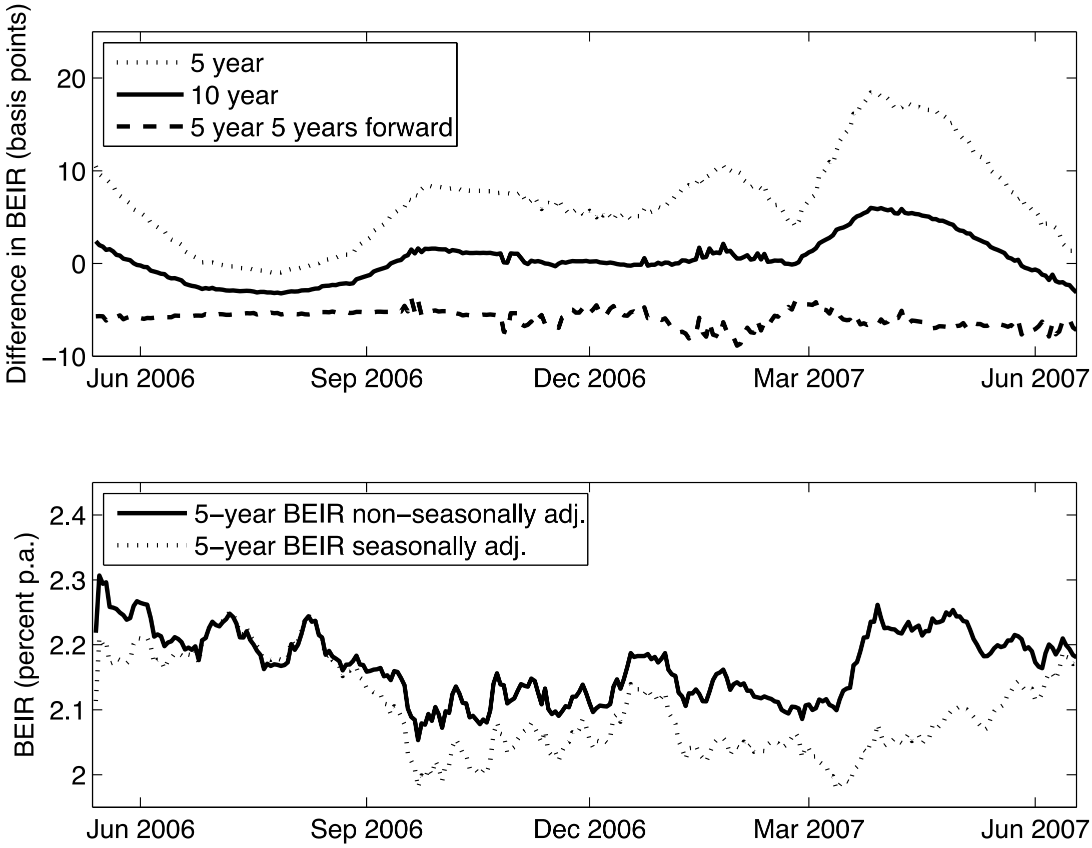

Figure 8: Top panel: Difference between unadjusted and seasonally adjusted BEIRs (a positive value implies that the unadjusted BEIR is higher). Lower panel: 5-year BEIRs in levels.

As shown in Figure 9, the dynamics of the inflation expectation measures extracted from swaps match the seasonally adjusted BEIRs much more closely that their seasonally unadjusted counterparts. For example, the correlation (in levels, from June 2006 to June 2007) between 3-year inflation swap rates and seasonally-adjusted BEIRs is 0.96, while it is only 0.77 in the case of the unadjusted rates. Moreover, in terms of their levels, the comparison of the seasonally-adjusted BEIR and the inflation swap rate reveals a relatively constant spread of about 10 basis points both for the 3-year and the 5-year maturities. The spread between break-even inflation rates derived from inflation-linked bonds and zero-coupon inflation swap rates is similar to the spread be-

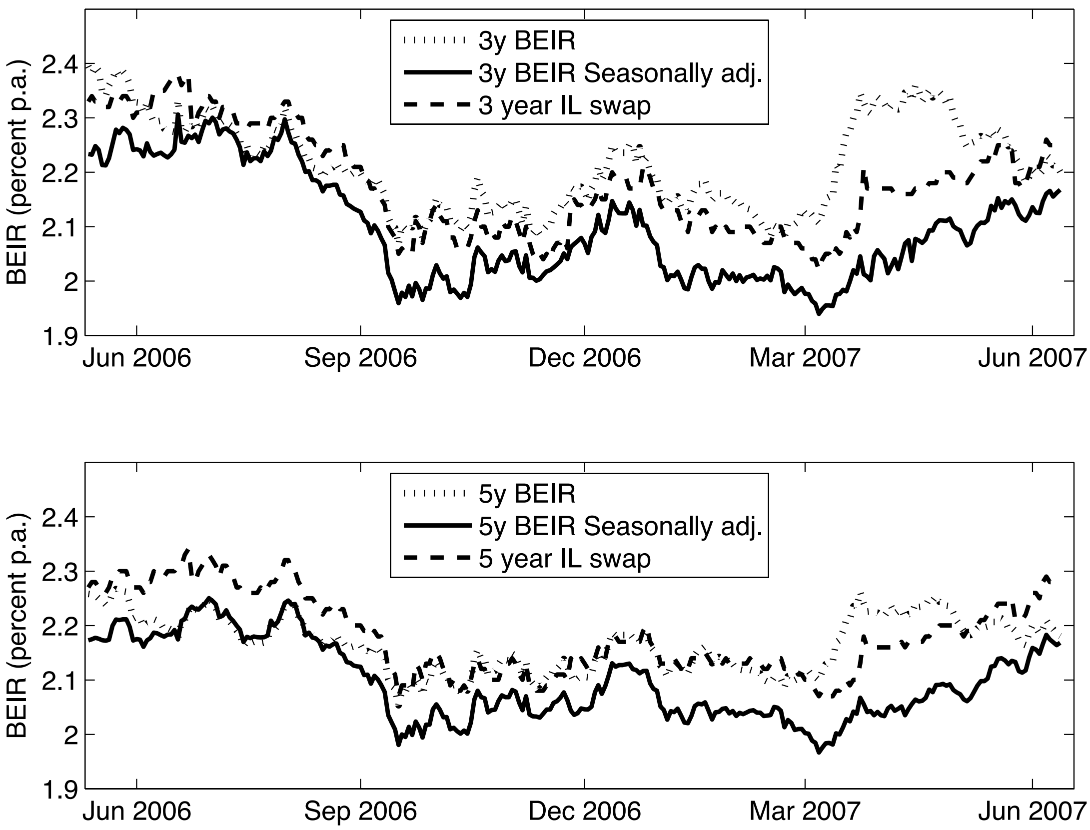

Figure 9: Unadjusted and seasonally adjusted 3-year and 5-year BEIRs and corresponding inflation swap rates.

tween the nominal government curve and the nominal swap curve. Counterparty risk in the swap market is most probably higher than default risk in the AAA-rated government bond market. Additionally, inflation swaps provide investors with the possibility to hedge inflation for a specific period, e.g. full three years from today. In principle, such a hedge could also be generated by a portfolio of inflation-linked bonds. In reality, however, constructing a reasonably precise replicating portfolio remains unfeasible given the still rather few available index-linked bonds. This could well imply that investors buying inflation protection may be willing to pay a premium for inflation swap contracts, thereby inducing a premium in observed inflation swap rates.

5 Concluding remarks

We have presented a methodology for estimating term structures of break-even inflation rates combining established methods for yield curve fitting with a novel procedure for obtaining seasonally adjusted yields and BEIRs.

We have shown that the impact of inflation seasonality on real yields and BEIRs are large enough to distort significantly the information content of BEIRs, in particular at short-to-medium maturities. Therefore, the proposed methodology for correcting the term structure of real and BEIRs for those effects should be relevant for anyone interested in the information provided by inflation-linked bonds, from policymakers to market participants actively involved in the trading of those instruments.

We have also shown, as a cross-check of the effectiveness of our approach, that the co-movement between BEIRs extracted from inflation-linked bonds and inflation swap rates becomes much stronger when the former are adjusted for seasonality. This is in line with intuition as inflation swap rates are, at least in principle, unaffected by seasonality because they refer to full-year maturities.

References

BIS (2005): "Zero-coupon yield curves: technical documentation," Bank for International Settlements, BIS Papers No 25.

Canty, P. (2007): "Seasonally adjusted prices for inflation-linked bonds," *Unpublished working paper, ABN Amro.*

D'Amico, S., D. Kim, and M. Wei (2007): "Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices," Federal Reserve Board, Working Paper.

ECB (2000): "Seasonal Adjustment of Monetary Aggregates and HICP for the euro area," European Central Bank.

Evans, M. D. (1998): "Real Rates, Expected Inflation, and Inflation Risk Premia," *Journal of Finance*, 53(1), 187–218.

Gapen, M. T. (2003): "Seasonal indexation bias in US Treasury Inflation-indexed Securities," *Applied Financial Economics*, 13, 509–516–218.

García, J. A., and A. van Rixtel (2007): "Inflation-linked bonds from a central bank perspective," ECB Occasional Paper series No 62.

Gurkaynak, R., B. Sack, and J. Wright (2007): "The U.S. Treasury yield curve: 1961 to the present," *Journal of Monetary Economics*, Forthcoming.

Nelson, C., and A. Siegel (1987): "Parsimonious Modeling of Yield Curves," *Journal of Business*, 60(4), 473–489.

Sack, B. (2000): "Deriving Inflation Expectations from Nominal and Inflation-Indexed Treasury Yields," Board of Governors of the Federal Reserve System, Working Paper.

——— (2002): "A Monetary Policy Rule based on Nominal and Inflation-Indexed Treasury Yields," Board of Governors of the Federal Reserve System, Working Paper.

Sack, B., and R. Elsasser (2004): "Treasury Inflation-Linked Debt: A Review of the U.S. Experience," *Economic Policy Review*, pp. 47–63, Federal Reserve Bank of New York.

Appendix

A Construction of daily price indices and seasonal factors

The purpose of this appendix is to define how the official releases on the HICP (ex. tobacco) are related to the lagged price indices, P_t^{Lag} and P_{t+h}^{Lag}, used in the paper. This requires some auxiliary definitions: first, let I_{M_t,Y_t} denote the actual monthly (unrevised) release towards the end of the month M_t+1. Note that due to the reporting lag, I_{M_t,Y_t} will typically first be released about six weeks after the date it is taken to refer to. In what follows, the subscript t will refer to a calendar date, and $\{D_t, M_t, Y_t\}$ refers to the day, month and year corresponding to this date. As an intermediate step, define the non-lagged daily price index as the following linear interpolation between two subsequent releases[25]

$$P_t = I_{M_t,Y_t} + \frac{D_t - 1}{days\ in\ M_t}\left[I_{M_t+1,Y_t} - I_{M_t,Y_t}\right] \tag{11}$$

The non-lagged daily price index is the basis for defining the lagged daily price index, which is for the indexation of the cash-flows of inflation-indexed bonds. We will denote this lagged index (sometimes called the 'reference price index') as P_t^{Lag}, and it is obtained by simply replacing the I's in (11) by their values three months back

$$P_t^{Lag} = I_{M_t-3,Y_t} + \frac{D_t - 1}{days\ in\ M_t}\left[I_{M_t-2,Y_t} - I_{M_t-3,Y_t}\right]$$

Note that the fraction just before the square brackets remains unaffected by the lag operator. This ensures that the interpolation also works for the lagged index on days such as 30 May, where the interpolation is done between February and March figures (3 month before May), despite that there are never 30 days in February. This definition is equivalent to the one used in practice by euro area sovereign issuers[26].

[25] If $M_t = 12$, of course, the interpolation will have to be between I_{M_t,Y_t} and I_{M_t-11,Y_t+1}. Note also that the year subscript have been left out in the main text for simplicity of notation.

[26] By applying the formula here to the preliminary euro area HICP (ex. tobacco) releases, one can thus exactly replicate the indexation figures published by e.g. the French treasury at www.aft.gouv.fr. Note also that the order of the lag operation and the interpolation is not interchangeable, due to the variation in numbers of days in each month

Consider the following numerical example, illustrating the calculation of P_t^{Lag} on 30 May 2007. For this we need $I_{FEB,2007} = 102.70$ and $I_{MAR,2007} = 103.39$ to calculate

$$P_t^{Lag} = 102.70 + \frac{30-1}{31}\left[103.39 - 102.70\right] = 103.34548$$

To obtain the applicable daily seasonal adjustment ($SF_t^{Lag}/SF_{t+h}^{Lag}$), there is a parallel need to interpolate between monthly values of the seasonal factors as well. The procedure is analogous to the interpolation between price index releases, but with the change that the monthly seasonal factors used in the interpolation are those of the previous year (for reasons explained in the paper)

$$SF_t^{Lag} = SFI_{M_t-3,Y_t} + \frac{D_t - 1}{days\ in\ M_t}\left[SFI_{M_t-2,Y_t} - SFI_{M_t-3,Y_t}\right]$$

where the SFI's are the monthly, multiplicative seasonal factors, which can be computed from publicly available data on HICP (ex. tobacco) and seasonally adjusted HICP (ex. tobacco).

B Seasonal adjustment in the presence of semi-annual coupons

Since there are currently no AAA-rated bonds with semi-annual coupons linked to euro area HICP, we have only included bonds with annual coupons in our empirical implementation. The proposed methodology for seasonally adjusting inflation-linked bond prices can, however, be slightly modified to also accommodate bonds with semi-annual coupons. In the case of semi-annual coupons the adjustment cannot be done directly by multiplying the (dirty) market prices by the ratio of seasonal factors (as in equation 10). Instead, exploiting that the coupon bond can be treated as a portfolio of zero-coupon bonds, a given bond's cash-flows may be split up into two parts: one containing the principal and coupons with the same seasonal as the principal, and another containing the remaining coupons (with a different seasonal). The model-implied prices of each of these sub-portfolios are then simply divided by the appropriate ratio of seasonal factors before being fitted to (unadjusted!) dirty market prices. Formally, we have

$$B_{t,i} + A_{t,i} = PV_{t,i}^1 + PV_{t,i}^2 \tag{12}$$

where

$$PV_{t,i}^1 = \frac{SF_{t+H_i^1}^{Lag}}{SF_t^{Lag}} C_i \sum_{h=1}^{H_i^1} I_{t,i}^1(h) e^{-yh^*} + e^{-yH_i^{*1}} \tag{13}$$

denotes the seasonally adjusted, model-implied price of the principal and same-seasonal (i.e. equal to seasonal of the maturity H^1) coupons,[27] and

$$PV_{t,i}^2 = \frac{SF_{t+H_i^2}^{Lag}}{SF_t^{Lag}} C_i \sum_{h=1}^{H_i^2} I_{t,i}^2(h) e^{-yh^*} \tag{14}$$

denotes the analogous price of the remaining coupons with the different seasonal matching the maturity H^2 of the last of such coupons).

C Estimating real yield curves adjusted for the indexation lag

In the main part of the paper we abstracted from the complication that the price index ratio P_{t+h}^{Lag}/P_t^{Lag} used to define the discount factor of an inflation-linked bond does not exactly correspond to the holding period t to $t+H$ of the bond. Consequentially, the traded bond will reflect information about the expectation of $\tilde{P}_t/P_t^{Lag}$, which corresponds to past, yet partially unknown, inflation. At the same time, an inflation-linked bond gives inflation compensation only up to time $t+H-l$, were l is the indexation lag.

[27] The indicator function $I^1(h)$ takes the value one if a cash-flow with same seasonal as the principal is due at time h, and zero otherwise.

Let $Q_t^{\widetilde{IL}}(h^*)$ denote the nominal price at time t of an inflation-linked zero-coupon bond paying $P_{t+H}^{Lag}/\tilde{P}_t$ at maturity, H calendar days hence. P_t^{Lag} is the value of the price index used for the indexation of inflation-linked bonds, and $\tilde{P}_t$ is the 'real-time' price level. When defining $\tilde{P}_t$, it appears most natural to let the official data releases refer to the middle of the reference month, rather than to the beginning of it. In terms of the definitions in Appendix A, this implies that approximately $\tilde{P}_t = P_{t-15}$.[28]

How can we estimate the discount factor adjusted for the indexation lag $Q_t^{\widetilde{IL}}(h^*)$ using inflation-linked coupon bonds? As mentioned above, the prices of inflation-linked coupon bonds are quoted as "clean prices" with respect to both accrued interest and accrued inflation. Therefore, the theoretical price of an inflation-linked coupon bond is given by[29]

$$[B_t(H) + AI_t] \frac{P_t^{Lag}}{P_0} = \frac{\tilde{P}_t}{P_0} C \left[\sum_{h=1}^{H} I_t(h) Q_t^{\widetilde{IL}}(h^*) + Q_t^{\widetilde{IL}}(H^*) \right] \qquad (15)$$

where the term $\tilde{P}_t/P_0$ captures the fact that the nominal cash-flows from the actual bonds reflect accrued inflation relative to the base price index P_0, and not just relative to $\tilde{P}_t$ as in our definition of the inflation-linked zero-coupon bonds adjusted for the indexation lag given in this section.[30]

Applying equation (15) to market prices of coupon bonds $i = 1, ..., I$ and adding measurement errors, the inflation-linked yield curve adjusted for the indexation lag can

[28]The definition of $Q_t^{\widetilde{IL}}(h^*)$ differs from the definition of $Q_t^{IL}(h^*)$ exactly by the ratio $\tilde{P}_t/P_t^{Lag}$ which is a consequence of the indexation lag.

[29]We ignore the fact that the nearest cash-flows may be less than l days (indexation lag) into the future. In this case the cash-flow should be valued using the discount factor of a nominal bond because no protection from future inflation is involved. However, it can be shown that the effect of using the inflation-linked discount function for such nominally risk-free cash-flows is negligible.

[30]The reason why our definition of a theoretical inflation-linked bond differs in this way from the bonds actually traded is to simply facilitate the derivation of purely forward-looking inflation expectations. The difference is purely definitional and does not involve additional assumptions.

be fitted using the methodology described in Section 3.2

$$[B_{t,i} + AI_{t,i}] \frac{P_t^{Lag}}{\tilde{P}_t} = C_i \sum_{h=1}^{H_i} I_{t,i}(h) \widetilde{Q_t^{IL}}(h^*) + \widetilde{Q_t^{IL}}(H_i^*) + \epsilon_{t,i}.$$

To arrive at this expression, we have implicitly assumed that the price index $\tilde{P}_t$ is available in real time. However, the euro area HICP for the current month is usually released only in the second half of the following month. Implementing the estimation method described above requires therefore an estimate of $\tilde{P}_t$. The fact that "real-time" inflation ($\tilde{P}_t$) is unknown (in contrast to (P_t^{Lag}) is the main practical obstacle to computing inflation-linked real rates and BEIRs adjusted for the indexation lag. Moreover, compared to the impact of seasonality, the impact of adjusting for the indexation lag (barring exceptional situations of strong spikes in inflation over the last three months) is likely to be modest.

C.1 Computing break-even inflation rates adjusted for the indexation lag

The discount factor for inflation-linked bonds $\widetilde{Q_t^{IL}}(h^*)$ adjusted for the indexation lag translates straightforwardly into a corresponding zero-coupon (log) yield $\widetilde{y_t^{IL}}(h^*)$ by

$$\widetilde{y_t^{IL}}(h^*) = -\frac{1}{h^*} \widetilde{Q_t^{IL}}(h^*).$$

Some further adjustments are needed, though, to compute zero-coupon BEIRs which refer exactly to the period starting at t and ending at $t + H$. The reason for this can be sketched by considering the definition of $\widetilde{Q_t^{IL}}(h^*)$ as the nominal price of an inflation-linked zero-coupon bond paying $P_{t+H}^{Lag}/\tilde{P}_t$ in period $t + H$. The inflation compensation stops l days before time H.

Neglecting risk premia, the yield $\widetilde{y_t^{IL}}(H^* + l^*)$ on an inflation-linked bond of maturity $H + l$ can be decomposed into the following weighted average of a truly real interest rate $y_t^*(H^*)$ of maturity H and a nominal forward rate[31]

$$y_t^{\widetilde{IL}}(H^*+l^*) = \frac{H}{H+l} y_t^*(H^*) + \frac{l}{H+l}\left[\frac{(H+l)y_t(H^*+l^*) - Hy_t(H^*)}{(H+l)-H}\right]. \qquad (16)$$

The decomposition relies on the fact that an inflation-linked bond provides with inflation compensation up to time H and the yield on this bond should therefore include a real interest rate of maturity H. But at the end of the bond's life, no inflation compensation is paid for a period, whose length equals the indexation-lag l . Therefore, the return over this period corresponds simply to a nominal forward rate (last term in equation 16).

A similar decomposition of a (nominal) interest rate for maturity $H+l$ into an interest rate for maturity H and a forward interest rate for the period H to $H+l$ is given by

$$y_t(H^*+l^*) = \frac{H}{H+l} y_t(H^*) + \frac{l}{H+l}\left[\frac{(H+l)y_t(H^*+l^*) - Hy_t(H^*)}{(H+l)-H}\right]. \qquad (17)$$

Subtracting equation 17 from equation 16 eliminates the forward interest term and after some rearrangements we get

$$y_t(H^*) - y_t^*(H^*) = \frac{H+l}{H}\left[y_t(H^*+l^*) - y_t^{\widetilde{IL}}(H^*+l^*)\right]. \qquad (18)$$

Neglecting risk premia, the difference between a nominal interest rate of maturity H and the corresponding real interest rate (left-hand side of equation 18) is the inflation expectation over the period t to H.

[31]This decomposition was proved by Evans (1998).

European Central Bank Working Paper Series

For a complete list of Working Papers published by the ECB, please visit the ECB's website (http://www.ecb.europa.eu).

790 "Asset prices, exchange rates and the current account" by M. Fratzscher, L. Juvenal and L. Sarno, August 2007.

791 "Inquiries on dynamics of transition economy convergence in a two-country model" by J. Brůha and J. Podpiera, August 2007.

792 "Euro area market reactions to the monetary developments press release" by J. Coffinet and S. Gouteron, August 2007.

793 "Structural econometric approach to bidding in the main refinancing operations of the Eurosystem" by N. Cassola, C. Ewerhart and C. Morana, August 2007.

794 "(Un)naturally low? Sequential Monte Carlo tracking of the US natural interest rate" by M. J. Lombardi and S. Sgherri, August 2007.

795 "Assessing the impact of a change in the composition of public spending: a DSGE approach" by R. Straub and I. Tchakarov, August 2007.

796 "The impact of exchange rate shocks on sectoral activity and prices in the euro area" by E. Hahn, August 2007.

797 "Joint estimation of the natural rate of interest, the natural rate of unemployment, expected inflation, and potential output" by L. Benati and G. Vitale, August 2007.

798 "The transmission of US cyclical developments to the rest of the world" by S. Dées and I. Vansteenkiste, August 2007.

799 "Monetary policy shocks in a two-sector open economy: an empirical study" by R. Llaudes, August 2007.

800 "Is the corporate bond market forward looking?" by J. Hilscher, August 2007.

801 "Uncovered interest parity at distant horizons: evidence on emerging economies & nonlinearities" by A. Mehl and L. Cappiello, August 2007.

802 "Investigating time-variation in the marginal predictive power of the yield spread" by L. Benati and C. Goodhart, August 2007.

803 "Optimal monetary policy in an estimated DSGE for the euro area" by S. Adjemian, M. Darracq Pariès and S. Moyen, August 2007.

804 "Growth accounting for the euro area: a structural approach" by T. Proietti and A. Musso, August 2007.

805 "The pricing of risk in European credit and corporate bond markets" by A. Berndt and I. Obreja, August 2007.

806 "State-dependency and firm-level optimization: a contribution to Calvo price staggering" by P. McAdam and A. Willman, August 2007.

807 "Cross-border lending contagion in multinational banks" by A. Derviz and J. Podpiera, September 2007.

808 "Model misspecification, the equilibrium natural interest rate and the equity premium" by O. Tristani, September 2007.

809 "Is the New Keynesian Phillips curve flat?" by K. Kuester, G. J. Müller and S. Stölting, September 2007.

810 "Inflation persistence: euro area and new EU Member States" by M. Franta, B. Saxa and K. Šmídková, September 2007.

811 "Instability and nonlinearity in the euro area Phillips curve" by A. Musso, L. Stracca and D. van Dijk, September 2007.

812 "The uncovered return parity condition" by L. Cappiello and R. A. De Santis, September 2007.

813 "The role of the exchange rate for adjustment in boom and bust episodes" by R. Martin, L. Schuknecht and I. Vansteenkiste, September 2007.

814 "Choice of currency in bond issuance and the international role of currencies" by N. Siegfried, E. Simeonova and C. Vespro, September 2007.

815 "Do international portfolio investors follow firms' foreign investment decisions?" by R. A. De Santis and P. Ehling, September 2007.

816 "The role of credit aggregates and asset prices in the transmission mechanism: a comparison between the euro area and the US" by S. Kaufmann and M. T. Valderrama, September 2007.

817 "Convergence and anchoring of yield curves in the euro area" by M. Ehrmann, M. Fratzscher, R. S. Gürkaynak and E. T. Swanson, October 2007.

818 "Is time ripe for price level path stability?" by V. Gaspar, F. Smets and D. Vestin, October 2007.

819 "Proximity and linkages among coalition participants: a new voting power measure applied to the International Monetary Fund" by J. Reynaud, C. Thimann and L. Gatarek, October 2007.

820 "What do we really know about fiscal sustainability in the EU? A panel data diagnostic" by A. Afonso and C. Rault, October 2007.

821 "Social value of public information: testing the limits to transparency" by M. Ehrmann and M. Fratzscher, October 2007.

822 "Exchange rate pass-through to trade prices: the role of non-linearities and asymmetries" by M. Bussière, October 2007.

823 "Modelling Ireland's exchange rates: from EMS to EMU" by D. Bond and M. J. Harrison and E. J. O'Brien, October 2007.

824 "Evolving U.S. monetary policy and the decline of inflation predictability" by L. Benati and P. Surico, October 2007.

825 "What can probability forecasts tell us about inflation risks?" by J. A. García and A. Manzanares, October 2007.

826 "Risk sharing, finance and institutions in international portfolios" by M. Fratzscher and J. Imbs, October 2007.

827 "How is real convergence driving nominal convergence in the new EU Member States?" by S. M. Lein-Rupprecht, M. A. León-Ledesma and C. Nerlich, November 2007.

828 "Potential output growth in several industrialised countries: a comparison" by C. Cahn and A. Saint-Guilhem, November 2007.

829 "Modelling inflation in China: a regional perspective" by A Mehrotra, T. Peltonen and A. Santos Rivera, November 2007.

830 "The term structure of euro area break-even inflation rates: the impact of seasonality" by J. Ejsing, J. A. García and T. Werner, November 2007.

ISSN 1561-0810
9 771561 081005