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IMPERFECT PREDICTABILITY AND MUTUAL FUND DYNAMICS

HOW MANAGERS USE PREDICTORS IN CHANGING SYSTEMATIC RISK

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EUROSYSTEM

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In 2008 all ECB publications feature a motif taken from the €10 banknote.

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CONTENTS

Abstract

Suppose a fund manager uses predictors in changing portfolio allocations over time. How does predictability translate into portfolio decisions? To answer this question we derive a new model within the Bayesian framework, where managers are assumed to modulate the systematic risk in part by observing how the benchmark returns are related to some set of imperfect predictors, and in part on the basis of their own information set. In this portfolio allocation process, managers concern themselves with the potential benefits arising from the market timing generated by benchmark predictors and by private information. In doing this, we impose a structure on fund returns, betas, and benchmark returns that help to analyse how managers really use predictors in changing investments over time. The main findings of our empirical work are that beta dynamics are significantly affected by economic variables, even though managers do not care about benchmark sensitivities towards the predictors in choosing their instrument exposure, and that persistence and leverage effects play a key role as well. Conditional market timing is virtually absent, if not negative, over the period 1990-2005. However such anomalous negative timing ability is offset by the leverage effect, which in turn leads to an increase in mutual fund extra performance.

Keywords: Equity mutual funds; conditional asset pricing models; time-varying beta; Bayesian analysis
JEL Classification: C11, C13, G12, G13.

Non-technical summary

This paper investigates how mutual fund managers use common predictors and private information in adjusting the systematic risk of their portfolios. Many recent papers have found that predictability of stock and bond returns should be reflected in investment decisions, but at an empirical level it is not yet clear how managers really use predictors in changing their portfolio structure over time.

In this paper we try to understand this issue. We first derive a theoretical framework of fund dynamics and then inspect, by means of econometric methods, a comprehensive sample of US domestic equity mutual funds over the period 1980-2005.

In our view, managers change their systematic risk exposure by observing how benchmark returns are related to a set of predictors, and on the basis of their own information set, which is stochastically inaccessible for external observers. Hence, predictors enter into the beta specification as imperfectly correlated covariates, and then deliver the beta up to a stochastic component.

Applying this logic, managers are assumed to take into account the potential benefits arising from market timing (via benchmark predictors), and from private information. This framework can accommodate two extreme cases: the first case is when predictors have no role and then the beta is a pure stochastic process. The second extreme case is when predictors capture the beta perfectly, as in classical conditional asset pricing literature. Our idea is to envisage a mechanism between these two extremes, since we are convinced that this approach could be helpful in inspecting how managers change their portfolio allocation depending on imperfect predictors and stochastic signals.

Within a Bayesian framework, we propose a new asset pricing model which combines time-variation, a stochastic component, and a deterministic component in the beta process, using a predictive system given by the portfolio excess return, the time-varying beta and benchmark excess return equations. Predictors are assumed to be imperfect and innovations in the system are correlated according to some data-theoretical priors. The system also delivers a measure for conditional market timing which is different from that of traditional conditional asset pricing models, because we accommodate imperfect predictors and beliefs about correlations among innovations.

Our empirical analysis has generated interesting results. First, the beta dynamics appear to be significantly affected by trend and term spread, which seem to be the most important predictor as instrument-based rules in beta variations, even though managers do not care about benchmark sensitivities towards the predictors in choosing their instrument exposure. Second, we detected average persistence in beta variation which is dependent both on fund styles and predictors. Again, long-run beta and persistent parameter have significant negative correlation, indicating that the lower the long-run beta, the higher the persistence then reflected in weak mean-reversion. In other terms, it may be possible that dynamic funds with significant beta variation exhibit low long-run beta average. Third, even if our Bayesian conditional measure revealed that no mutual fund category showed significant timing ability over the period 1990-2005, we found significant leverage effect in portfolio returns. This result is of particular interest since we noted that when the effect is negative, the anomalous negative market timing is offset by a positive correlation between portfolio and benchmark innovations, leading to an increase in the extra performance of mutual funds.

I. Introduction

Predictability of excess returns and mutual fund abnormal performance are two central topics in finance. On the one hand, there is abundant empirical evidence proving that aggregate stock and bond returns are predictable by some informative variables. On the other hand, contrasting evidence appears in the study of predictability of abnormal fund performance. Jang, Yao and Yu (2007) note on this issue that although many studies prove that aggregate market returns are predictable, and such predictability should translate into optimal asset allocation, it is difficult – even for sophisticated and informed investors – to take advantage of such predictability in portfolio decisions. Again, exploring the link between variable selection and portfolio choice, Ait-Sahalia and Brandt (2001) point out that although stock and bond returns are partly predictable, it has proven difficult to translate the evidence of return predictability into practical portfolio advice. This is probably the reason why, at empirical level, predictability of security returns does not translate into predictability of fund performance. Indeed, it is not yet clear how managers *really* use predictors of stock and bond returns in changing their portfolio structure over time.

Recent finance literature addressed the issue of how managers *should* use theory and data in forming portfolio allocation. Campbell and Thompson (2007) prove that imposing some weak theoretical-based restriction on the sign of the coefficients is economically meaningful for mean-variance investors. Wachter and Warusawitharana (2007) find that predictability in the data is sufficient to influence investments, even for investors with a high degree of prior scepticism. Again, other related Bayesian studies such as Kandel and Stambaugh (1996), Stambaugh (1999), Barberis (2000), Pastor (2000) and Avramov (2004) show that return predictability has a large impact on portfolio choice.

One intriguing question is how to reconcile what managers should do with what managers really do. This is our objective. To this end we inspected a large sample of US domestic equity mutual funds over the period 1980-2005, exploring different and converging issues.

First, in order to estimate time variation in a portfolio's risk loading, we introduce an asset pricing model conceived with the aim of combining time-variation and a stochastic component in the beta process and conditional pricing models. Within a Bayesian framework, the fund returns are modelled by imposing a pseudo-stochastic process on the path of risk loading. Indeed, we allow the betas to vary over time according to (a) a set of observable instruments, and (b) a set of unknown factors that evolve following a stochastic process. In such a model managers reallocate their assets on the basis of some partly-unknown factors to be estimated through the state space technology. On this point, our methodology is similar to that of Mamaysky, Spiegel and Zhang (2007), who estimate alphas and betas with the Kalman filter. There is also an analogy with Jostova and Philipov (2005), where a stochastic mean-reversion structure beta is introduced by allowing the coefficient to be expressed as a process that joins time-variation with a stochastic

component. We extend this approach by combining (i) a time-variation, (ii) a stochastic component, and (iii) a deterministic component in the beta process.

Second, as in Pastor and Stambaugh (2007), we propose a framework in which the predictors are assumed to be imperfect and innovations in the predictive system are correlated according to some data-theoretical priors. Indeed, by imposing a structure on fund returns, betas and benchmark returns, we develop a framework that could help examine how managers really use predictors in changing investments over time. Such methodology is also useful to assess the timing ability within a conditional asset pricing model context in the spirit of Ferson and Schadt (1996) and Becker, Ferson, Myers and Schill (1999). However, the way we measure market timing is different because we accommodate imperfect predictors and beliefs about correlations among innovations. In a sense, we introduce a Bayesian Conditional Market Timing approach by imposing a correlation structure among the innovations of portfolio returns, time-varying betas and benchmark returns.

The main finding of our empirical analysis is that beta dynamics are significantly affected by economic variables that act as imperfect benchmark predictors. The trend and term spread seem to be the most important predictors as instrument-based rules in beta variations, even though managers do not care about benchmark sensitivities towards the predictors in choosing their instrument exposure, either in sign or in magnitude. We also found average persistence in beta variation, though some significant differences occur for specific fund styles also depending on predictors. Interestingly, long-run beta and persistent parameter have significant negative correlation, indicating that the lower the long-run beta, the higher the persistence then reflected in weak mean-reversion. This leads us to conclude that dynamic funds with significant beta variation exhibit low long-run beta average. As for market timing, our Bayesian conditional measure revealed that no mutual fund category showed significant timing ability over the period 1990-2005, although the Info Tech Sector could be viewed as a very near to significant market timer, getting positive correlation between beta and benchmark innovations at little more than 0.1 level. Finally, we found significant leverage effect in portfolio returns. Indeed, we noted that when the effect is negative, the anomalous negative market timing is offset by a positive correlation between portfolio and benchmark innovations, leading to an increase in the extra performance of mutual funds.

The remainder of the paper proceeds as follows. Section 2 introduces the theoretical framework of fund dynamics and describes our model. Section 3 presents the estimation procedure, discussing how the base ingredients of our Bayesian approach are selected. Section 4 presents the data. Section 5 reports the empirical analysis on mutual fund performance and beta dynamics, while Section 6 inspects conditional market timing. Section 7 looks in more depth at the beta dynamics by exploring the variance decomposition. Section 8 concludes.

II. Fund Dynamics

The design of portfolio returns follows a parsimonious three-equation representation in the spirit of Admati, Bhattacharya, Pfleiderer and Ross (1986). The system contains the portfolio return equation for a given fund, the time-varying benchmark risk exposure and the benchmark return forecasting model. The assumption is that managers are single-period investors who maximise the conditional expectation of an increasing, concave objective function that depends on returns in excess of the risk-free rate, $r_p = R_p - R_f$ where R_p denotes the return of the managed portfolio and R_f the return of the risk-free asset. The expectation is conditional on a set of instrumental variables, Z_t, and a Gaussian private signal about the future performance of the market, S_t. As in Becker, et al. (1999), the maximisation problem is simplified by assuming that portfolio managers choose between the risky market portfolio with return R_m and the risk-free asset. Hence, $R_p = R_f + wR_m$ where w is the weight of the market portfolio. In this setting, the portfolio choice problem is

$$(1) \qquad \max_{w_t} E\big[u\big(r_{p,t}\big)\big|Z_t, S_t\big].$$

The solution of the manager's problem is to adjust the market exposure w according to Z and S every t period, assuming the objective function to be time-invariant, namely the conditional distribution of the returns $r_{p,t}$ given the predictors, and the signal is time homogenous. Formally,

$$(2) \qquad w_t = w\big(Z_t, S_t\big)$$

which implies that optimal market portfolio weight in t depends on the conditional mean-variance ratio of the tangency portfolio, given the information Z and S in t.[1]

As pointed out by Chen, Ferson and Peters (2006), since w evolves over time depending linearly on the private signal, the return of the managed portfolio with market timing ability should exhibit a convex pattern relative to the benchmark return. Classical market timing approaches gave convincing theoretical reasons for such functional form, which can be expressed by a quadratic function of the market returns (Treynor, Mazuy, 1966) or an "isomorphic correspondence" to some non-linear option strategies pay-off (Merton, 1981). However, this convexity hypothesis is a critical point in finance. Indeed, old and new empirical evidence is contradictory and often shows anomalous concavity. Other problems arise also between selectivity and market timing. Glosten and Jagannathan (1994) pointed out that spurious market timing may be latent in the negative correlation between the two components if managers take long put option positions, which move down the beta portfolio when stock returns are low.

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[1] Becker *et al.* (1999) discuss the way to find optimal weights in such a setting, which require first order condition of the maximisation problem using a constant Rubinstein-type measure of risk aversion (Rubinstein, 1973).

Again, Grinblatt and Titman (1989) highlighted problems in performance measurement when a fund's beta varies without any active portfolio rebalancing. It is thus clear that beta modelling play a key role in making equations (1) and (2) the theoretical framework in which fund dynamics can be scrutinised.

This study introduces a novel approach to achieving this goal, enabling us to explain how expected and unexpected market returns are related to expected and unexpected portfolio returns. In such a framework, the systematic risk exposure is the unobservable fulcrum of a system in which benchmark and fund returns are connected through economic imperfect predictors.

II.1. The Model

The econometric representation of the model we propose generalises the conditional asset pricing approach by assuming that predictors enter into the beta specification as imperfect correlated covariates. This signifies that they are correlated to the systematic risk dynamics and that they can or cannot deliver the beta perfectly, depending on the size of the stochastic component. On the one hand, we have one extreme possibility in which there are no predictors, collapsing the model for beta as a pure stochastic process. On the other hand, we have a second extreme possibility when predictors capture the expected beta perfectly, as in classical conditional asset pricing literature. Our model lies between these two extremes, as we are convinced that such intermediate view could be helpful in describing the fund dynamics and assessing the true ability of a manager in forecasting future market returns.

Specifically, our framework combines the following three equations:

$$(3) \qquad r_{p,t} = \alpha_p + \beta_{p,t} r_{m,t} + \varepsilon_{p,t}$$

$$(4) \qquad (1 - \phi L)(\beta_{p,t} - \mu) = \Gamma' \mathbf{z}_{t-1} + \eta_{p,t}$$

$$(5) \qquad r_{m,t} = \Lambda' \mathbf{z}_{t-1} + u_{m,t}.$$

Equation (3) is the excess portfolio returns over the risk-free rate at any time t, where α_p denotes the risk-adjusted abnormal return, $\beta_{p,t}$ the systematic risk exposure of the portfolio assumed to be time varying, $r_{m,t}$ the excess market return over the risk-free rate and $\varepsilon_{p,t}$ the unexpected portfolio return.

The equation (4) is the time-varying beta, where L denotes the lag operator, ϕ the persistence beta parameter, μ the unconditional mean reverting beta term, Γ' the transposed vector of

sensitivities, $\mathbf{z}_{t-1}$ the vector of predictors at time $t-1$, and $\eta_{p,t}$ the beta stochastic component to accommodate imperfect predictors in beta evolution[2], which can also be viewed as a noisy private signal about future market return in a way that will be clear further on. Note that our beta specification allows for different beta-generating processes as in Jostova and Philipov (2005) while introducing a structural component in the beta variation to be linear in the state variables (the term $\Gamma'\mathbf{z}_{t-1}$). Indeed, if $\phi \equiv 1$ the process is a unit root, i.e. a random walk with shocks in betas that persist indefinitely, and if $\phi \equiv 0$ the process is a perfect mean reversion. Consider also that since ϕ is the shrinkage parameter towards the long-run beta μ, stationarity in beta and returns is guaranteed by imposing $|\phi| < 1$.

Equation (5) is the excess market returns over the risk-free rate, where $\Lambda'\mathbf{z}_{t-1}$ denotes the expected market return at time t modelled as a linear function of the same predictors in (4), with Λ' denoting the transposed vector of sensitivities and $\mathbf{z}_{t-1}$ the vector of predictors at time $t-1$, and $u_{m,t}$ is the unexpected market return at time t, then accommodating imperfect predictors.

To combine equations (3), (4) and (5), we impose a structure on the system innovations. Indeed, it seems plausible to believe that superimposed on the residuals there may be a non-negative covariance matrix whose off-diagonal elements could arise as a result of a market timing ability as well as leverage effects on beta. Then, the assumption is that the system innovations exhibit the following i.i.d. distribution:

$$
(6) \qquad \begin{bmatrix} \varepsilon_{p,t} \\ \eta_{p,t} \\ u_{m,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} & \sigma_{\varepsilon u} \\ \sigma_{\eta\varepsilon} & \sigma_\eta^2 & \sigma_{\eta u} \\ \sigma_{u\varepsilon} & \sigma_{u\eta} & \sigma_u^2 \end{bmatrix} \right).
$$

Equations (3)-(4)-(5) together with assumption (6) form the core of this paper and require some economic and technical discussions.

II.2. Economic Rationale of the Model

To clarify the economic framework of our system, we reformulate equation (3) by using (4) and (5). We assume also that innovations of (3) and (4) are continuous and differentiable functions of beta and benchmark innovations to denote with $G(\eta)$ and $F(u)$ respectively. Furthermore, to

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[2] In contrast, conditional asset pricing models as in Ferson, Schadt (1996) consider a predictive regression approach in which the linear combination of lagged predictors assumes that the true conditional expected beta is explained perfectly, i.e. without error, by observed predictors.

allow for market timing ability we admit a simple quadratic form of $F(u)$, where the second order term captures additional[3] convexity in returns of the managed portfolio with respect to the benchmark returns. To simplify the model, we do not impose a similar structure on $G(\eta)$, since the only effect we would model in this way would be the leverage effect in the beta process then resulting in a linear function of η innovations. In this setting we have:

$$r_{p,t} = \alpha_p + \left(\beta_{p,t-1} + \Gamma' \mathbf{z}_{t-1} + \eta_{p,t}\right) r_{m,t} + \varepsilon_{p,t}$$

with

$$\eta = F(u)$$
$$\varepsilon = G(\eta)$$

Our interest is in the relation between r_p and r_m. To examine this issue, a second-order Taylor series expansion of δr_p is used to assess how a small change in r_m translates into r_p:

(7)
$$
\begin{aligned}
\delta r_p &= \frac{dr_p}{dr_m} \delta r_m + \frac{1}{2} \frac{d^2 r_p}{dr_m^2} \delta r_m^2 \\
&= \left[\mu + \phi\left(\beta_{p,t-1} - \mu\right) + \Gamma' \mathbf{z}_{t-1} + \eta_t + \frac{d\eta}{dr_m} r_m + \frac{d\varepsilon}{dr_m} \right] \delta r_m + \left[\frac{d\eta}{dr_m} + \frac{1}{2} \frac{d^2\eta}{dr_m^2} + \frac{1}{2} \frac{d^2\varepsilon}{dr_m^2} \right] \delta r_m^2 \\
&= \left[\mu + \phi\left(\beta_{p,t-1} - \mu\right) + \frac{d\eta}{dr_m}\left(\Lambda' \mathbf{z}_{t-1} + u_{m,t}\right) + \frac{d\varepsilon}{dr_m} \right] \delta r_m + \left[\frac{d\eta}{dr_m} + \frac{1}{2} \frac{d^2\eta}{dr_m^2} + \frac{1}{2} \frac{d^2\varepsilon}{dr_m^2} \right] \delta r_m^2
\end{aligned}
$$

Since we assume that η is a continuous second-order differentiable function of the benchmark innovation, then a possible specification could be $\eta = \tilde{\eta}\sigma_\eta + au + bu^2$, where $\tilde{\eta}$ is the idiosyncratic Gaussian beta stochastic component, σ_η is the conditional idiosyncratic beta volatility, while a and b are parameters that measure, respectively, the linear and the convex relation with benchmark surprises. Such assumption leads to the simple results $\dfrac{d\eta}{dr_m} \equiv \dfrac{d\eta}{du}$ and

$\dfrac{d^2\eta}{dr_m^2} \equiv \dfrac{d^2\eta}{du^2}$.

---

[3] As compared to the convexity induced by using predictability in conditioning variables.

In the same way, since ε is a continuous first-order differentiable function of beta innovations, an admissible representation could be $\varepsilon = \tilde{\varepsilon}\sigma_\varepsilon + c\eta$, where $\tilde{\varepsilon}$ is the pure Gaussian portfolio return stochastic component, σ_ε is the idiosyncratic portfolio return volatility, while c is the sensitivity towards the unexpected beta variation. Then, $\dfrac{d\varepsilon}{dr_m} \equiv \dfrac{d\eta}{dr_m}c \to \dfrac{d\eta}{du}c$. These arguments lead to the following representation:

(8)
$$
\begin{aligned}
\delta r_p &= \left[\mu + \phi(\beta_{p,t-1} - \mu) + \eta_t + \frac{d\eta}{du}r_m + \frac{d\eta}{du}c\right]\delta r_m + \left[\frac{d\eta}{du} + \frac{1}{2}\frac{d^2\eta}{du^2} + \frac{1}{2}\frac{d^2\eta}{du^2}c\right]\delta r_m^2 \\
&= \left\{\mu + \phi(\beta_{p,t-1} - \mu) + \eta_t + \frac{d\eta}{du}\left[(\Lambda' \mathbf{z}_{t-1} + u_{m,t}) + c\right]\right\}\delta r_m + \left[\frac{d\eta}{du} + \frac{1}{2}\frac{d^2\eta}{du^2}(1+c)\right]\delta r_m^2,
\end{aligned}
$$

which identifies the key factors in explaining the relationship between r_p and r_m. As one can note, the response of portfolio return to variation in r_m is a function of:

  i)   the sensitivities towards the predictors for both the beta and the benchmark return, Γ and Λ;
  ii) the volatility scaled idiosyncratic Gaussian beta stochastic component as well as the possible linear and quadratic relation of beta innovations with the benchmark surprises, namely the process for η_t and,
  iii) the potential leverage effect as measured by the parameter c, which represents the linear response of unexpected portfolio return to the unexpected beta variation.

Consider the point i), i.e. how r_p is related to r_m through the predictor variables. As noted, the key variables are the vector sensitivities of beta and benchmark return. How should the beta move relative to the benchmark process and how should Γ be linked to Λ? Answering these questions is analogous to finding the dependence between portfolio adjustments and market return signals, indeed, calculating the potential benefit of market timing exploitable using predictability. As shown by Cochrane (1999), this can be achieved simply by referring to the explanatory power of the market return equation, which in turn is closely related to the difference between the squared Sharpe ratio of a manager who conditions on predictor variables and the unconditional squared Sharpe ratio. By denoting such a difference with ΔSR^2 and with R^2 (the percent variance of the market return explained by the predictors), it is easy to show that:

(9)
$$
R^2 = \frac{\Delta SR^2}{\Delta SR^2 + 1}
$$

and equivalently,

$$\Delta SR^2 = \frac{R^2}{1-R^2};$$

(10)

$$\Delta SR = \sqrt{\frac{R^2}{1-R^2}}$$

If we assume, for example, an R^2 of 5 per cent, the gain arising from timing investments based on values of the predictors is $\Delta SR = 0.2294$; if instead the R^2 is supposed to be very high, say 90 per cent, the corresponding ΔSR is 3. As is evident, the higher the explanatory power, the higher the gain from changing portfolio composition based on market return signals. However, since the empirical literature proves that predictability is quite tenuous with R^2s that seldom exceed 10 per cent, ΔSR is not as high as it might be in theory. Furthermore, other doubts arise from taking mechanical market timing rules as suggested by the predictors:

- First, parameter uncertainty could affect asset allocation decisions by lowering the expected benefit from market timing. This is clear considering the variance of the predictors and the variance of the benchmark noise. To make the point easier, consider having only one predictor, so $r_{m,t} = \lambda_1 z_{1,t-1} + u_{m,t}$. If R^2 is low, σ_u^2 is large enough with respect to $\lambda_1^2 \sigma_z^2$. In this case, also having low values of σ_z^2, which raise λ_1, the market timing gains exploitable through the market signal $\lambda_1 z_{1,t-1}$ is modest relative to the benchmark noise. Again, further complications arise when there are many predictors considered simultaneously, since the covariance matrix becomes extremely relevant for potential gains from timing investments.
- Second, as argued by Campbell and Thompson (2007) if expected excess return is negative, one should disregard the estimates of Λ in choosing the portfolio weights, which in turn reflects on Γ.

Taken together, these reasons explain why the sign and the magnitude of beta sensitivities could differ from those of the benchmark return for investors who try to solve the problem denoted by equation (1).

Point ii) is about the connection between innovations of beta and benchmark processes. The mechanism we have in mind is an alternative way to inspect the market timing ability in the spirit of conditional asset pricing models. Given that the model for beta allows for predictability, the stochastic component gives direct information about the manager's private signals implied in the

time varying systematic risk exposure. The idea is similar to that of Becker, et al. (1999), where using the Heinkel and Stoughton (1995) approach to model the private signal, the authors distinguish timing ability that merely reflects publicly available information, as captured by a set of instrumental variables, from conditional market timing based on better information. Having the same objective, we try to separate the true market timing ability by removing the spurious quantity related to the public information, focusing on the stochastic component of the systematic risk. As one can note by observing equation (8), if the quality of the private information signal is good, we will observe $\frac{d\eta}{du} > 0$ which indicates that the manager has increasing information about the true process for benchmark return as $\frac{d\eta}{du}$ becomes large. Furthermore, $\frac{d^2\eta}{du^2}$ also plays a key role: having a value greater than zero is equivalent to adding more convexity in the function of r_p relative to r_m.

Finally, point iii) introduces possible leverage effect through the correlation between the unexplained portfolio return with the stochastic beta component. Admitting ε to be a continuous first-order differentiable function of beta innovations is equivalent to linking the portfolio's return innovations to the benchmark surprises "passing through the systematic risk". As noted, within this structure, the response of ε to r_m reduces to $\frac{d\eta}{du}$ time the sensitivity towards the unexpected beta variation, expressed by the parameter c in equation (8). Since this parameter enters into the first and the second order approximation, the leverage effect could influence linear as well as quadratic r_p variations.[4]

Considered together, arguments i)-ii)-iii) produce a complex function for r_p where the effects are mixed with each other and difficult to inspect. To make the point easier, let us consider a market timer who uses public as well as private information for benchmark returns. The S&P 500 return in excess of the 1 month T-bill over the period January 1990 – December 2005 is the benchmark excess return to be estimated by equation (5). Suppose, further, the lagged state variables, demeaned and standardised, are: default spread, dividend yield, illiquidity, interest rates, stock market volatility, and term spread.[5] The view is that predictors are used, first, to make estimates about the future value of the benchmark return, and, second, to adjust the systematic risk exposure of the portfolio. In doing so, the main assumption is that the manager is concerned with the explanatory power of the state variables as well as the sign and the magnitude of the Λ

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[4] As noted in Jostova and Philipov (2005), while leverage effects are present in time-varying stochastic volatility, Braun, Nelson, and Sunier (1995) do not find such effects in the beta process. Similarly, Jostova and Philipov in their paper do not find any leverage effects for either single-security or portfolio betas. In our model, imposing $c = 0$ is analogous to assuming absence of leverage effect.

[5] See Section III.2 for details about these instruments and the way we measured them.

vector. Afterward, he or she will decide the systematic risk exposure towards the state variables Γ by considering the expected excess return, then using superior information. Hence, the manager focuses on the sign and the magnitude of expected and unexpected benchmark excess return. This is equivalent to assuming that beta variation follows market or private signals depending both on the sign and on the magnitude of the two. If, indeed, the benchmark shock is greater than the expected excess return, it should be more profitable for a perfect market timer to follow the private signal; otherwise, the signal to be included in the beta variation should be that of the market. Obviously, when expected and unexpected market returns have the same sign both the signals will enter into the beta variation.

We now inspect the response of the portfolio's return using the system (3)-(4)-(5). To this end we start estimating equation (5) over the period 01/90-12/05. The R-squared is near 11 per cent [6] and the vector of sensitivities is $\Lambda' = \begin{bmatrix} -0.0049 & 0.0182 & 0.0144 & -0.0224 & 0.0059 & -0.0141 \end{bmatrix}$, all the values significant at 0.1 level. In our setting, the manager is supposed to use public and private information about the benchmark excess return so as to maximise the conditional Sharpe ratio. We suppose, also, that (a) the mean reverting beta term is $\mu = 1$; (b) the starting beta is $\beta_{p,t-1} = 1$; (c) the manager constrains the beta variation to be expressed in terms of maximum monthly standard deviation of 20 per cent with $\pm$ 2 per cent bounds (hence $0.18 \leq \Delta\beta \leq 0.22$). With this objective function[7], it is now interesting to note how the correlation between stochastic beta and unexpected benchmark component ($\rho_{\eta u}$) impacts on the behaviour of r_p together with expected beta variation. To this end, letting $\rho_{\eta u}$ 0.9 we parametrically generated the stochastic beta using $\eta = \tilde{\eta}\sigma_\eta + au + bu^2$ assuming for simplicity $\sigma_\eta = 0$ and supposing the variance of the beta variation explained by predictors to be 10 per cent. With these numbers, the perfect market timer who maximises the conditional Sharpe ratio subject to the beta constraint will have $\Gamma' = \begin{bmatrix} -0.0179 & 0.0780 & 0.0633 & -0.1069 & -0.0063 & -0.0588 \end{bmatrix}$. It is noteworthy that in the single-period constrained maximisation process[8], the magnitude of Γ became different with respect to Λ, and the sign for the trend variable was negative while the sign is positive for the benchmark excess return process. This is because of parameter uncertainty and negative expected excess returns which imply mechanical market timing rules as suggested by the predictors to be modified in order to maximise the conditional Sharpe ratio.

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[6] The corresponding adjusted R-squared is 8.61 per cent.

[7] In essence, the problem is a maximisation of simple quadratic objective function, leading to the well-known maximised expected utility given by $u[w(Z_t, S_t)] = \dfrac{1}{2\tau} E(SR|Z,S)^2$, where τ is the relative risk aversion and $E(SR|Z,S)^2$ is the expected Sharpe ratio of the ex ante tangency portfolio of the risky asset conditional on Z and S.

[8] As noted in Section II, we assumed that managers are single-period investors who maximise the conditional expectation of an increasing, concave objective function that depends on returns in excess of the risk-free rate.

In this numerical exercise we also considered the leverage effect, since it could play a critical role in performance pattern. To inspect this, we referred to the parametric expression of unexplained portfolio return using the expression $\varepsilon = \tilde{\varepsilon}\sigma_\varepsilon + c\eta$ and assuming $\sigma_\varepsilon = 0$; $\rho_{\varepsilon\eta} = 0.9$ with R-squared of equation (3) equal to 0.9 (i.e. modest leverage effect). Figure 1 represents case a), which is the manager who does not exhibit the leverage effect, and case b), where the return pattern of the manager has also a positive leverage effect. Note that in both cases the maximisation problem leads to a convex pattern of portfolio excess return, which increases in magnitude by considering the leverage effect.

Figure 1

Portfolio Excess Return Pattern for Perfect Market Timer
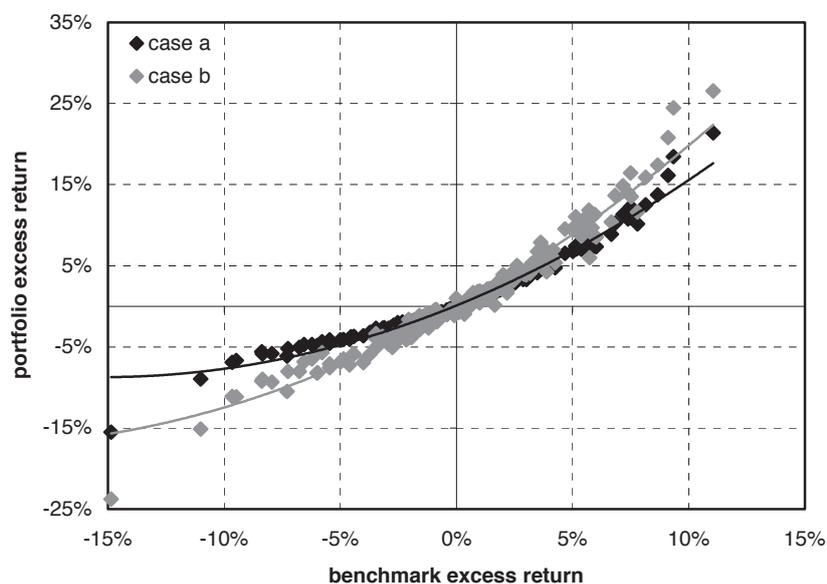
III. Estimation Approach

To estimate our three-equation system we developed a Bayesian approach within a state space technology, hence treating the parameters of the model as random variables. Under Bayesian analysis, one starts by having some initial ideas about these unknowns, to be represented by a probability distribution over all the possible values. Afterward, one collects data to improve this understanding and obtain the parameter estimates by combining beliefs with data. Hence, priors and likelihood are the basic ingredients to obtain a posterior, which summarises the information embedded in returns regarding latent state variables and parameters. More precisely, the Bayesian

approach, first, specifies a joint prior distribution; second, identifies the likelihood function; third, computes the joint posterior distribution of the parameters and the data.

Therefore, let us start by denoting with $\boldsymbol{\theta} = [\Sigma, \alpha, \phi, \mu, \Gamma, \Lambda]$ the parameters of the system where Σ is the covariance matrix of the system innovations, i.e.

$$\Sigma = V\left(\begin{bmatrix} \varepsilon_{p,t} \\ \eta_{p,t} \\ u_{m,t} \end{bmatrix}\right) = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} & \sigma_{\varepsilon u} \\ \sigma_{\eta\varepsilon} & \sigma_\eta^2 & \sigma_{\eta u} \\ \sigma_{u\varepsilon} & \sigma_{u\eta} & \sigma_u^2 \end{bmatrix}.$$

Let us further denote by $p(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta})$ the joint prior distribution, in which the values for $\beta_{p,t}$ are modelled as in equation (4), and by L the likelihood function expressed as[9]

(11)
$$\mathsf{L} = p(\mathbf{r}_{pT}, \mathbf{r}_{mT} | \boldsymbol{\theta}) \equiv \int p(\mathbf{r}_{pT}, \mathbf{r}_{mT} | \boldsymbol{\theta}, \boldsymbol{\beta}_{p,T}) p(\boldsymbol{\beta}_{p,T} | \boldsymbol{\theta}) d\boldsymbol{\beta}_{p,T}$$
$$\boldsymbol{\beta}_{p,T} = [\beta_{p,0}, \beta_{p,1}, ..., \beta_{p,T}],$$
$$\mathbf{r}_{pT} = [r_{p,1}, r_{p,2}, ..., r_{p,T}],$$
$$\mathbf{r}_{mT} = [r_{m,1}, r_{p,2}, ..., r_{m,T}],$$

where $\beta_{p,t}$ are obtained through a simulation procedure that uses the Kalman filter.

Theoretically, the joint posterior distribution of parameters and latent variables (the betas) is

(12)
$$p(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta} | \mathbf{r}_{pT}, \mathbf{r}_{mT}) \propto p(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta}) p(\mathbf{r}_{pT}, \mathbf{r}_{mT} | \boldsymbol{\theta}, \beta_{p,t}).$$

In our paper this posterior distribution is simulated using a Gibbs sampling-data augmentation procedure, a Markov Chain Monte Carlo (MCMC) technique that generates random samples from a given target distribution, namely the joint posterior distribution of the parameters $\boldsymbol{\theta}$ and the state variables given the observed returns $p(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta} | \mathbf{r}_{pT}, \mathbf{r}_{mT})$. The details are in Appendix A.

As pointed out by Johannes and Polson (2007), the Clifford-Hammersley theorem gives a formal motivation to the construction of the MCMC algorithms, stating that a joint distribution can be characterised by its so-called complete conditional distributions.

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[9] In order to (slightly) simplify notation, here we leave dependence on the variables z completely implicit. This is perfectly legitimate since these variables are assumed exogenous.

Strictly speaking, our model is a state space system in which equations (3) and (4) are, respectively, the measurement and the transition (or state) equations, and where equation (5) characterises the benchmark evolution based upon lagged predictors. It is just this equation that complicates the system. Indeed, two critical points arise in our model. The first is connected to the fact that r_m is endogenously specified (by equation (5)) and enters into the measurement equation, while the second refers to the model parameters, which interact in a non-linear way. In order to handle these complexities, our Bayesian approach is developed according to the procedure detailed in Appendix A.

III.1. Priors

As discussed in the previous section, within a Bayesian setting prior distributions translate into posterior distribution in light of data, and this as an implication of the laws of probability. We emphasise this point because such a technicality plays a key role in the understanding process. In other words, prior distributions allow the researcher to incorporate possibly non-sample or extra-sample information in a consistent manner. To do this, in our paper we assumed the role of econometricians who use empirical evidence in a "pre- (or extra-) sample" with two objectives: first, forming initial ideas about the values of the unknown parameters, and second, modulating the dogmatic-sceptical range with which the priors are transformed into posterior belief.

Analytically, we assume that our view or information on coefficients within the processes for beta and benchmark depends on what we learned in the pre-sample, while for the extra performance we assume a zero Jensen's alpha with a large value for the variance, reflecting a weak prior for the manager's excess return. Such a prior for alpha implies a "semi-strong market efficiency" view across the managers, which can be viewed as a sort of dependence across funds in the spirit of Jones and Shanken (2005). Then, our prior for alpha is normally distributed, cross-sectional centred towards zero with a certain variance. Partly different, the economic logic underlying the priors for beta and benchmark coefficients, as well as the covariances of the system shocks, is completely empirical for both the first and second moment of the prior distributions. As external returns observers, we look at the data making prior estimates of the system then deriving the posterior by modulating the shrinkage towards the priors according to the model reliability, as measured by the R-squared of the regressions in the pre-sample. In this way, the priors can be diffuse or dogmatic on the basis of the empirical evidence of the extra-sample. This is the linkage between the economic motivation of our model and the technology used by the estimation approach. As discussed in Section II.2, the R-squared of the benchmark equation is strictly connected to the potential benefit of market timing exploitable using predictability. Note that, on this issue, we argued how such a benefit can be approximated by the difference between the squared Sharpe ratio of a manager who conditions on predictor variables and the unconditional squared Sharpe ratio, i.e. $\Delta SR^2 = R^2 / (1 - R^2)$. The point is of particular

interest, because such formulation recalls the recommended shrinkage factor proposed by Connor (1997) to estimate the slope coefficient of the expected market return. Indeed, Connor (1997) suggests imposing an informative prior centred on the economic notion of (weak form) market efficiency, which implies that the slope coefficient should be zero. To this end the posterior Bayesian slope coefficient is the OLS estimate scaled by a shrinkage factor that depends on the relative precisions of the OLS estimate and the prior mean, which is given by $\left\{T / \left[T + \left(R^2 / 1 - R^2\right)^{-1}\right]\right\}$. Then, the R-squared modulates the dogmatic-sceptical view in making estimates. And this is particularly appealing in our model, since ΔSR^2 simultaneously play a key role both in measuring the benefit of predictability in market timing and in modulating the shrinkage towards the priors.

To obtain the priors for the beta process, we first relied on the approach of Ferson and Schadt (1996), in which $r_{p,t}$ is linearly modulated on a time-varying conditional beta times the market return net of the short term instrument, $r_{m,t}$, supposing the beta of the CAPM to be a function of some set of public informational lagged variables, Z_{t-1}. Formally:

$$(13) \qquad r_{p,t} = \alpha_p + \beta_p\left(Z_{t-1}\right)r_{m,t} + e_{p,t}$$

$$(14) \qquad \beta_p\left(Z_{t-1}\right) = b_{0p} + B'_p z_{t-1}$$

where $z_{t-1} = Z_{t-1} - E(Z)$ is a vector of the deviations of Z_{t-1} from the unconditional means, B'_p is the vector with a dimension equal to the dimension of Z_{t-1} and whose elements measure the response of the conditional beta to the information variables, and b_{0p} is the average (or mean reverting equivalent) beta parameter. Slightly modifying the original approach of Ferson and Schadt (1996), we directly used the standardised Z_{t-1} vector, in order to derive scale-independent coefficient estimates. As is evident, in this predictive regression approach the linear combination of lagged predictors are assumed to deliver the true conditional expected beta perfectly. On the other hand, in our setting we deal with imperfect predictors and this introduces a stochastic component that potentially could also completely dominate the predictability in Z_{t-1}. The fact that we admit a noisy predictability in the beta process suggested to us a two-pass procedure in forming the priors. Indeed, first we run regressions (13)-(14) obtaining the time-varying beta estimates $\hat{\beta}_p\left(Z_{t-1}\right)$, and second, we use these estimates to run the following equation,

$$(15) \qquad \hat{\beta}_{p,t}\left(Z_{t-1}\right) = \kappa_{0p} + \varphi_p\left(\hat{\beta}_{p,t-1}\left(Z_{t-1}\right) - \kappa_{0p}\right) + K'_p z_{t-1} + \xi_{p,t}.$$

In this way we derive the priors for the mean reverting beta term κ_{0p}, the persistent parameter φ_p, and the vector of sensitivities K'_p, and where the corresponding second moments are derived from the estimated parameters covariance matrix in equation (15). Finally, the variances for K'_p can be shrunk using the factor $\left\{ T_0 \big/ \left[T_0 + \left(R^2 / 1 - R^2 \right)^{-1} \right] \right\}$ in which the R-squared is that of the benchmark equation using the same instruments with T_0 to denote the number of observations in the pre-sample.[10]

The logic of our system is now clear. Indeed, the central reasoning focuses on the assumption that managers modulate the systematic risk of their portfolios in part by observing how the benchmark returns are related to some predictors, and in part on the basis of their own informational set, which is stochastically inaccessible for econometricians who basically observe only the return patterns over time. It is for this reason that the same set of instruments enters both in the beta process and in the benchmark equation. Again, in changing the portfolio composition, hence implying a variation in beta, the managers take into account the potential benefit arising from the market timing exploitable by the benchmark predictors and by the private information. This explains why the beta process is characterised by two components, one deterministic and one stochastic. Furthermore, managers could be more confident in following common or private signals based upon their own investment philosophy, and in this regard, they have a measure of the potential market timing benefit, which is linked to the R-squared of the benchmark equation, when using common instruments, and to the covariances among the shocks of portfolio returns, the beta variations and the benchmark returns, when using private information.

To make this logic suitable for econometric applications, we then formed initial ideas about the potential values for the parameters by using the conditional pricing model *à la* Ferson-Schadt. Mean and variance of each parameter are then derived empirically by regression estimates, to be used in the Gibbs sampler in order to obtain the posterior. In doing this, the dogmatic-sceptical view of the manager about the benchmark predictability is indeed our view in making estimates of beta parameters. In more depth, the way we have to shrink towards the priors is set by the model reliability for the benchmark, whose R-squared modulates the confidence in the predictors. If they have poor forecasting power, they deliver little market timing benefits and it is therefore plausible to assume that the manager could revise the portfolio composition possibly giving little weight to such predictors. Hence, the factor ΔSR^2 is not merely important for technical reasons, since it also plays a fundamental economic role in translating priors into posteriors.

In synthesis, prior distributions for the parameters of the model are constructed as follows:

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[10] In this version of the paper we report only the results obtained when not applying this adjustment to the prior variance. We are currently carrying out a more careful assessment of the evaluation of the R-squared-based shrinkage.

i) Given the initial sample $t = 1,2,..,T_0$ we estimate equation (13) by OLS to obtain estimates to be plugged into (15) which, in turn, is estimated by OLS. The resulting OLS estimates and the estimated covariance matrix are then used as first and second moments for a multivariate Gaussian distribution for the parameters of equation (4).

ii) Pre-sample estimates of the betas are also used to estimate the intercept in equation (3). Its point estimate and its variance are used as moments for a univariate Gaussian prior for α_p.

iii) OLS on pre-sample period is also used to estimate equation (5). OLS point estimate of Λ and its OLS covariance matrix are used as moments for a multivariate Gaussian prior.

iv) Pre-sample OLS estimates of equations (3), (4) and (5) generate residuals $\hat{\boldsymbol{\eta}}_t, t = 1,2,...,T_0$. The sample covariance matrix of these residuals is then used to calibrate a Wishart prior on the inverse of the covariance matrix $\boldsymbol{\Sigma}$ in equation (6), i.e. the covariance matrix of the errors in the system, as follows

$$\boldsymbol{\Sigma}^{-1} \sim Wishart(\underline{\mathbf{v}}, \underline{\mathbf{S}}),$$

(16)

$$\underline{\mathbf{v}} = \frac{T_0}{4}, \underline{\mathbf{S}} = \left(\frac{4}{T_0} \sum_{t=1}^{T_0} \hat{\boldsymbol{\eta}}_t \hat{\boldsymbol{\eta}}_t^{'} \right)^{-1}.$$

In this way we account for the likely scale of those errors without imposing too tight a prior.

III.2. Instruments for Beta Process

The literature on predictability of equity returns using lagged values of economic and financial variables is extremely extensive. Among the many studies on this issue, and more precisely focused on the performance measurement conditional on economic states, the work of Ferson and Qian (2004) is our main reference. In revisiting the conditional asset pricing model in measuring the performance of US equity mutual funds, they select eleven potential instruments for the states of the economy to be used in conditioning the performance of mutual funds. These are (1) the level of short-term interest rates, measured as the bid yield to maturity on a 90-day Treasury bill; (2) the term structure slope (or, simply, term spread), measured as the difference between a five-year and a one-month discount Treasury yield (other works used the yield difference between the ten and one-year government bonds); (3) the term structure concavity, approximated by $y_3 - (y_1 - y_5)/2$ where y_j is a j-year fixed-maturity yield; (4) the interest rate volatility, which is the monthly standard deviation of three-month Treasury rates, computed from the days within that month and controlled for autocorrelation in spot rates; (5) the stock market volatility, constructed using daily returns for the Standard and Poor (S&P) 500 index as for the interest rate volatility; (6) the credit (or default) spread, computed as the yield difference between Moody's BAA and AAA-rated corporate bonds; (7) the dividend yield, computed as the annual dividend

yield of the CRSP value-weighted stock index (other works used the sum of dividends paid on the S&P index over the past 12 months divided by the current level of the index); (8) inflation, computed as the percentage change in the consumer price index, CPI-U; (9) the industrial production growth, which is the monthly growth rate of the seasonally-adjusted industrial production index; (10) short-term corporate illiquidity, which is the percentage spread of three-month high-grade commercial paper rates over three-month Treasury rates; (11) the stock market liquidity, as measured by Pastor and Stambaugh (2003), which is based on price reversals. To extend the list of possible predictors we can consider an additional instrument suggested by a reading of Ait-Sahalia and Brandt (2001) which is (12) the trend, computed as the difference between the log of the current S&P index level and the log of the average index level over the previous 12 months.

Among these instruments, we selected two sets of potential predictors. The first is that used in Ait-Sahalia and Brandt (2001), namely the default spread, the log dividend-to-price ratio of the S&P index, the term spread, and an S&P index trend (or momentum) variable. As noted by Ait-Sahalia and Brandt (2001), the economic rationale of this selection has to be referred to Fama and French (1988, 1989), who show that the first three predictors capture cyclical time variations in excess stock and bond returns, and Keim and Stambaugh (1986), who use a variable similar to trend in order to predict returns. For our paper, such a choice seems theoretically and methodologically consistent, since using these instruments Ait-Sahalia and Brandt (2001) show how to select and combine variables to best predict an investor's optimal portfolio weights. Consider further that the same authors in their paper perform preliminary regressions to verify if predictors capture time variations in the first and second moments of excess bond and stock returns. And after having demeaned and standardised the data, their GMM estimates prove that all the four variables are significant predictors.

The second set of potential predictors was selected by a pure statistical variable selection approach. In more depth, among the above list of 12 instruments we selected those predictors that maximise the adjusted R-squared for the first and the second sub-periods simultaneously, and imposed a constraint on the maximum number of predictors – four – in line with Ait-Sahalia and Brandt (2001). In doing this, we chose three variables: short-term corporate illiquidity, inflation, and the term spread.

III.3. Posterior simulation of the model

For the sake of clarity, let us write the key elements of the model here

$$r_{p,t} = \alpha_p + \beta_{p,t} r_{m,t} + \varepsilon_{p,t}$$

$$\beta_{p,t} = c + \phi \beta_{p,t-1} = \Gamma' \mathbf{z}_{t-1} + \eta_{p,t}, c = \mu(1-\phi)$$

$$r_{m,t} = \Lambda' \mathbf{z}_{t-1} + u_{m,t},$$

$$\begin{bmatrix} \varepsilon_{p,t} \\ \eta_{p,t} \\ u_{m,t} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} & \sigma_{\varepsilon u} \\ \sigma_{\eta\varepsilon} & \sigma_\eta^2 & \sigma_{\eta u} \\ \sigma_{u\varepsilon} & \sigma_{u\eta} & \sigma_u^2 \end{bmatrix} \right).$$

This is a Gaussian state space system. It is not linear owing to the multiplicative interaction between $\beta_{p,t}$ and $r_{m,t}$ in equation (3) but, via convenient step-wise simulation, the posterior distribution of parameters and latent variables can easily be simulated by means of a Gibbs sampling-data augmentation procedure. In this system we can distinguish five blocks:

1. the latent variables β_t;
2. the parameter in equation α_p;
3. the parameters in equation c, ϕ, γ;
4. the parameters in equation Λ;
5. the parameters in Σ.

Given the priors being implemented, each of these blocks is simulated from its conditional posterior distribution. Details on these conditional distributions and on how to simulate from them are given in Appendix A.

IV. Data

IV.1. Mutual funds, Benchmark and Predictors

The mutual fund sample consists of monthly returns of 5,337 open-ended US equity funds from January 1980 to December 2005, as supplied by the Center for Research in Security Prices (CRSP) Survivor-Bias Free U.S. Mutual Fund Database. The time period was split into two intervals, the first from January 1980 to December 1989 while the second from January 1990 to December 2005. We call the sub-sample of mutual fund returns within the first time period a "pre-sample" to be used for priors estimation, while the sub-sample over the second time period an "estimation sample" to be used in estimating our system. Mutual funds included in the sample have at least two years of data (48 monthly observations) in the period from January 1990 to December 2005, regardless of the return window location. Monthly returns are calculated as total returns, therefore reflecting the reinvestment of dividends and capital gains. In the empirical analysis we worked with equally weighted mutual funds portfolios (EWP) according to the ownership style category as provided by Standard and Poor's Style Name. The reason why we deal with EWPs is twofold. First, mutual fund benchmarks summarise the behaviour of

homogeneous classes of funds in terms of style investing, thus giving a "general" view of each specific style[11], which is particularly useful in identifying common characteristics of funds. Second, having portfolio returns over the entire period from January 1980 to December 2005 is suitable for robust estimation, and portfolios of mutual funds can obviously meet this requirement, although at the cost of losing money managers' idiosyncratic information. Analytically, we formed 17 EWPs, reported below in Table 1, in which we report the corresponding univariate descriptive statistics for the monthly returns, for both the pre-sample and the estimation sample. Table 2 and Figure 2 describe returns for market benchmark, risk free asset, and the first set of predictors. Panel A of Table 2 reports univariate descriptive statistics for Treasury bill, S&P index and the four predictors used in Ait-Sahalia and Brandt (2001). Panel B shows pairwise correlations of the predictors with: the predictors, the excess stock return and traditional market timing proxies given by Treynor-Mazuy square excess stock return and the Henriksson-Merton piecewise term for upward and downward excess stock return. Figure 2 plots autocorrelations and time-series for predictors over the period from January 1980 to December 2005.

In the same way, Table 3 and Figure 3 describe the second set of predictors. Panel A of Table 3 reports univariate descriptive statistics for illiquidity, inflation and term spread, while Panel B shows pairwise correlations of the predictors with: the predictors, the excess stock return and traditional market timing proxies. Figure 3 plots autocorrelations and time-series for illiquidity and inflation, since term spread is just depicted in Figure 2.

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[11] Consider, also, that instead of executing 5,605 simulations, when dealing with individual managers, we run the procedure for averages of mutual funds, thus reducing the very high computational burden of our simulation study.

Table 1: Descriptive Statistics of EWPs

	Mean	Median	Max	Min	StdDev	Skew	Kurtosis	ρ_1
			Pre-sample: from 1980/01 to 1989/12					
All Cap Growth	0.0126	0.0143	0.1380	-0.2388	0.0587	-0.6996	5.2100	0.1230
All Cap Value	0.0118	0.0111	0.1155	-0.1963	0.0408	-0.8411	7.8104	0.1081
Energy Sector	0.0123	0.0142	0.1509	-0.2779	0.0574	-1.1655	8.3371	0.0421
Financials Sector	0.0134	0.0132	0.1012	-0.2138	0.0445	-1.0855	7.4413	0.2292
Healthcare Sector	0.0125	0.0151	0.1253	-0.2531	0.0531	-0.9797	7.2769	0.0964
InfoTech Sector	0.0111	0.0111	0.1910	-0.2822	0.0647	-0.4535	5.9564	0.0526
Large Cap Blend	0.0129	0.0163	0.1127	-0.2073	0.0442	-0.9075	7.2705	0.0874
Large Cap Growth	0.0142	0.0150	0.1344	-0.2323	0.0503	-0.8656	6.9542	0.1008
Large Cap Value	0.0139	0.0134	0.1062	-0.1879	0.0400	-0.9645	7.5575	0.0987
Materials Sector	0.0162	0.0055	0.2834	-0.2907	0.1021	-0.0666	3.5830	0.0148
Mid Cap Blend	0.0123	0.0140	0.1108	-0.2253	0.0435	-1.5491	10.1126	0.0891
Mid Cap Growth	0.0141	0.0166	0.1500	-0.2603	0.0537	-0.9285	7.6786	0.1058
Mid Cap Value	0.0133	0.0141	0.1436	-0.2113	0.0492	-0.5303	6.1128	0.0754
Small Cap Blend	0.0138	0.0180	0.1613	-0.2688	0.0553	-1.0177	7.7613	0.1414
Small Cap Growth	0.0144	0.0182	0.1367	-0.2695	0.0558	-1.0354	7.6123	0.1586
Small Cap Value	0.0125	0.0187	0.1483	-0.2336	0.0548	-0.9434	6.6086	0.2072
Utilities Sector	0.0145	0.0150	0.1410	-0.0909	0.0354	0.1954	4.6455	0.0990
			Estimation Sample: from 1990/01 to 2005/12					
All Cap Growth	0.0104	0.0147	0.1796	-0.1864	0.0543	-0.2687	3.7526	0.0865
All Cap Value	0.0109	0.0138	0.0884	-0.1632	0.0385	-0.6928	4.7379	0.1136
Energy Sector	0.0100	0.0083	0.1859	-0.2060	0.0569	0.1933	4.5387	-0.0239
Financials Sector	0.0135	0.0206	0.1448	-0.2103	0.0475	-0.5703	5.0400	0.0780
Healthcare Sector	0.0128	0.0140	0.2247	-0.1660	0.0523	0.2579	4.7753	0.0607
InfoTech Sector	0.0145	0.0176	0.2792	-0.2772	0.0881	-0.2265	3.7749	0.0766
Large Cap Blend	0.0083	0.0121	0.1060	-0.1403	0.0380	-0.5143	3.8378	-0.0067
Large Cap Growth	0.0086	0.0111	0.1219	-0.1581	0.0462	-0.4803	3.6214	0.0374
Large Cap Value	0.0088	0.0123	0.1041	-0.1411	0.0370	-0.5236	4.2825	0.0118
Materials Sector	0.0075	0.0099	0.3430	-0.2062	0.0736	0.2808	4.7143	-0.0602
Mid Cap Blend	0.0108	0.0141	0.0971	-0.1759	0.0417	-0.6926	4.5624	0.0939
Mid Cap Growth	0.0106	0.0138	0.2129	-0.2001	0.0586	-0.2393	4.2383	0.0775
Mid Cap Value	0.0107	0.0141	0.1060	-0.1638	0.0384	-0.7119	5.1017	0.1123
Small Cap Blend	0.0103	0.0145	0.1261	-0.1899	0.0472	-0.6104	4.2442	0.1109
Small Cap Growth	0.0111	0.0172	0.2422	-0.2088	0.0637	-0.1723	4.0304	0.1008
Small Cap Value	0.0109	0.0181	0.0969	-0.1781	0.0422	-0.8028	4.8689	0.1945
Utilities Sector	0.0077	0.0079	0.0865	-0.1105	0.0327	-0.4765	3.7264	0.1074

The table shows monthly descriptive statistics of equally weighted style matched portfolios provided by Standard and Poor's Style Name 5,337 of open-ended US equity mutual funds over the periods from January 1980 to December 1989 and from January 1990 to December 2005.

Table 2: Returns and First Set of Predictors

	Mean	Median	Max	Min	StdDev	Skew	Kurtosis	ρ_1
Panel A: Descriptive Statistics								
Pre-sample: from 1979/12 to 1989/11								
T-Bill	0.0071	0.0064	0.0152	0.0028	0.0025	0.9707	3.5909	0.8490
S&P	0.0111	0.0127	0.1318	-0.2176	0.0473	-0.7760	6.8004	0.0441
Def	0.0149	0.0140	0.0266	0.0076	0.0045	0.7744	2.8355	0.8758
LnDP	1.4326	1.4573	1.8888	1.0039	0.2142	-0.0284	2.0655	0.9688
Trend	0.0602	0.0708	0.2451	-0.2207	0.0951	-0.4853	2.5820	0.8585
Term	0.0086	0.0130	0.0254	-0.0316	0.0119	-1.1313	3.8114	0.8904
Estimation Sample: from 1989/12 to 2005/11								
T-Bill	0.0033	0.0037	0.0068	0.0006	0.0015	-0.0679	2.3452	0.9544
S&P	0.0074	0.0101	0.1116	-0.1458	0.0412	-0.4479	3.6847	-0.0434
Def	0.0083	0.0079	0.0141	0.0053	0.0021	0.9613	3.1976	0.9358
LnDP	0.6931	0.5958	1.3814	0.0831	0.3535	0.0896	1.8260	0.9827
Trend	0.0420	0.0506	0.2352	-0.2636	0.0884	-0.8523	3.5918	0.8848
Term	0.0143	0.0114	0.0331	-0.0049	0.0105	0.1943	1.6827	0.9800

	Def	LnDP	Trend	Term	r_m	$(r_m)^2$	$I_{rm>0}(r_m)$	$I_{rm<0}(r_m)$
Panel B: Correlations								
Pre-sample: from 1979/12 to 1989/11								
Def	1.0000	0.7166	-0.1390	-0.1053	0.0728	0.0067	0.0998	0.0239
LnDP		1.0000	-0.3941	-0.2900	0.0415	-0.0890	0.0255	0.0428
Trend			1.0000	0.0188	-0.0485	-0.0221	-0.1033	0.0184
Term				1.0000	0.1276	-0.0112	0.0625	0.1468
Estimation Sample: from 1989/12 to 2005/11								
Def	1.0000	0.0422	-0.4797	0.3794	-0.0276	0.1037	0.0388	-0.0822
LnDP		1.0000	0.0176	0.3136	0.0956	-0.1434	-0.0106	0.1677
Trend			1.0000	-0.2546	0.0180	-0.2121	-0.0917	0.1163
Term				1.0000	-0.0210	-0.1398	-0.1154	0.0744

Panel A of this table shows monthly descriptive statistics for Treasury bill, S&P index and four predictors: the default spread Def, the log dividend-to-price ratio of the S&P index LnDP, the S&P index momentum variable Trend, and the term spread Term. Panel B shows correlations of the predictors with: the predictors, the excess stock return r_m its square $(r_m)^2$ and the Henriksson-Merton piece-wise term for upward and downward excess stock return, $I_{rm>0}(r_m)$ and $I_{rm<0}(r_m)$, namely the indicator variable for positive and negative excess stock return, $I_{rm>0}$ and $I_{rm<0}$, multiplied by the excess stock return (r_m).

Table 3: Second Set of Predictors

				Panel A: Descriptive Statistics				
	Mean	Median	Max	Min	StdDev	Skew	Kurtosis	ρ_1
			Pre-sample: from 1979/12 to 1989/11					
ILL	0.0087	0.0076	0.0299	0.0003	0.0055	1.1178	4.5608	0.6428
Infl	0.0042	0.0036	0.0152	-0.0046	0.0036	0.6495	3.9165	0.6758
Term	0.0086	0.0130	0.0254	-0.0316	0.0119	-1.1313	3.8114	0.8904
			Estimation Sample: from 1989/12 to 2005/11					
ILL	0.0037	0.0037	0.0117	0.0003	0.0022	0.8923	4.2871	0.7803
Infl	0.0024	0.0021	0.0122	-0.0080	0.0026	0.1709	4.9077	0.2983
Term	0.0143	0.0114	0.0331	-0.0049	0.0105	0.1943	1.6827	0.9800

			Panel B: Correlations				
			Pre-sample: from 1979/12 to 1989/11				
	ILL	Infl	Term	r_m	$(r_m)^2$	$I_{r_m>0}(r_m)$	$I_{r_m<0}(r_m)$
ILL	1	0.3876	-0.4827	-0.0021	0.1479	0.1343	-0.1287
Infl		1	-0.6051	-0.1906	0.0558	-0.1185	-0.1958
Term			1	0.1276	-0.0112	0.0625	0.1468
			Estimation Sample: from 1989/12 to 2005/11				
	ILL	Infl	Term	r_m	$(r_m)^2$	$I_{r_m>0}(r_m)$	$I_{r_m<0}(r_m)$
ILL	1	0.0293	-0.5394	0.1966	0.0369	0.2134	0.1229
Infl		1	-0.0451	-0.0612	-0.0218	-0.0677	-0.0371
Term			1	-0.0210	-0.1398	-0.1154	0.0744

Panel A of this table shows monthly descriptive statistics for three predictors: short-term illiquidity ILL, inflation Infl, and the term spread Term. Panel B shows correlations of the predictors with: the predictors, the excess stock return r_m its square $(r_m)^2$ and the Henriksson-Merton piece-wise term for upward and downward excess stock return, $I_{r_m>0}(r_m)$ and $I_{r_m<0}(r_m)$), namely the indicator variable for positive and negative excess stock return, $I_{r_m>0}$ and $I_{r_m<0}$, multiplied by the excess stock return (r_m).

Figure 2: First Set of Predictors
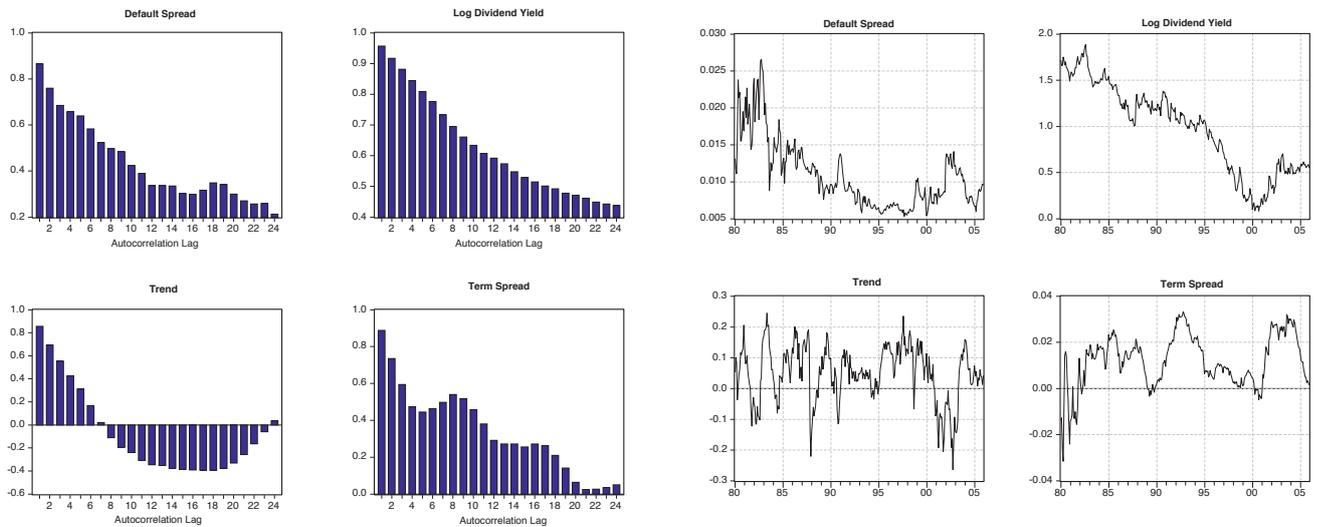
This figure shows autocorrelograms and time-series plots for the default spread, the log-dividend-to-price ratio of the S&P index, the S&P index trend variable, and the trend variable. The data cover the period from January 1980 through December 2005 and refer to monthly observations.

Figure 3: Second Set of Predictors
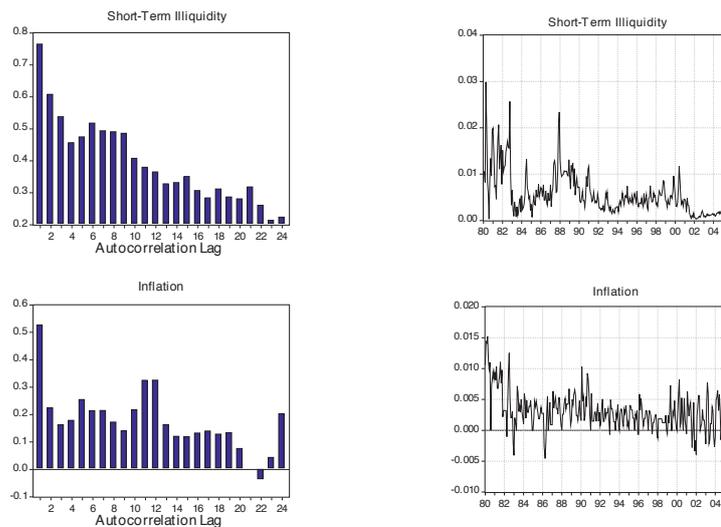
This Figure shows autocorrelograms and time-series plots for short-term illiquidity, inflation, and the term spread variable. The data cover the period from January 1980 through December 2005 and refer to monthly observations.

IV.2. Predictive Regressions

In order to verify the predictability of the selected instruments, we first estimated equation (5) using the first and second set of predictors demeaned and standardised. To control for heteroskedasticity and autocorrelation in covariance estimate, we used the Newey-West (1987) procedure. Table 4 presents the regression results. For the first set, the adjusted R^2 is negative for pre-sample, estimation sample, and overall sample. The term spread appears as the unique economically and statistically significant predictor but only for the period from 1980/01 to 1989/12. On the other hand, the adjusted R^2s for regressions using the second set of predictors are always positive and coefficients appear significant when considering the overall sample (the term spread is nearly significant at 0.1 level). For the pre-sample we observe that only inflation is a significant predictor, while for the estimation sample the most important predictor is illiquidity.

The first major fact to emerge from these regressions is that predictability arises only when illiquidity, inflation and term spread are considered, even if the explanatory power is very weak, according to the adjusted R^2 which is on average around 2.5 per cent. The second point is that sensitivity estimates for the first set of predictors appear extremely noisy, reflecting a sceptical view about predictability. In our model, this signifies less weight on priors for instruments of beta. Again, owing to the modest adjusted R^2 for both sets of predictors, the potential benefit of market timing exploitable using predictability appears extremely poor and the expectation is that managers do not mechanically follow timing rules as suggested by predictors. Indeed, Γ and Λ should differ in sign and magnitude.

Table 4: Benchmark Predictability

	First Set of Predictors					Second Set of Predictors			
	Def	LnDP	Trend	Term	Adj. R^2	ILL	Infl	Term	Adj. R^2
	Pre-sample: from 1980/01 to 1989/12								
coeff	0.0033	0.0009	-0.0016	0.0068	-0.0066	0.0049	-0.0093	0.0028	0.0212
p-value	0.6079	0.9002	0.6602	0.0655		0.4396	0.0924	0.6677	
	Estimation Sample: from 1990/12 to 2005/12								
coeff	-0.0006	0.0046	-0.0002	-0.0021	-0.0139	0.0108	-0.0026	0.0048	0.0326
p-value	0.8733	0.1818	0.9554	0.5458		0.0009	0.3025	0.1146	
	Overall Sample: from 1980/01 to 2005/11								
coeff	-0.0006	0.0032	0.0000	0.0017	-0.0116	0.0083	-0.0052	0.0047	0.0245
p-value	0.8389	0.2434	0.9835	0.4874		0.0093	0.0192	0.1080	

This table presents predictive regressions for expected excess S&P index using two sets of predictors. The first set contains four predictors: the default spread Def, the log dividend-to-price ratio of the S&P index LnDP, the S&P index momentum variable Trend, and the term spread Term. The second contains three predictors: short-term illiquidity ILL, inflation Infl, and the term spread Term. Estimates are computed using the Newey-West Heteroskedasticity and Autocorrelation Consistent (HAC) covariance estimates.

V. Mutual Fund Performance and Beta Dynamics

For each EWP we run the system (3)-(6) according to the prior elicitation as discussed in Section III.1 and using both sets of predictors. From equation (3), note that α is constant and expresses the Bayesian Jensen's alpha for EWPs. And since our beta is a process that joins mean reverting time-variation, where stochastic and deterministic components are both considered and where instruments are assumed to provide imperfect forecasts, we are able to deliver an unbiased estimate of excess returns. Consider, also, that beta dynamics gives direct information on market timing ability conditional on public information within a Bayesian context through $\sigma_{u\eta}$ in (6). In other words, we deliver a Bayesian Conditional Market Timing measure.

Tables 5 and 6 report the parameters estimated from the model and Table 7 shows correlations among the EWPs to inspect whether coefficients are, in some way, related. Note that in computing correlations for shocks in the system we only refer to off-diagonal elements, since they give information on various angles of mutual fund performance; the off-diagonal elements we used are not covariances but correlations, computed by using the parameter in (6).

In Table 5 we have posterior estimates when the default spread, the log-dividend-to-price ratio of the S&P index, the S&P index trend variable, and the trend variable are used. The Jensen's alpha

appears positive and statistically significant for 12 out of 17 EWPs. This is particularly interesting, since all the funds seem to deliver a positive extra performance when fund dynamics are inspected assuming imperfect predictability. The same conclusion holds when the short-term illiquidity, inflation, and the term spread are taken as predictors. Table 6 indeed indicates that 15 EWPs deliver positive and significant Jensen's alpha. Interestingly, in Table 7 we observe that when the first set of predictors is used, the correlation between alphas and $\mathrm{corr}(\eta, \varepsilon)$ is -0.5247 and reaches significance at 0.05 level. Since $\mathrm{corr}(\eta, \varepsilon)$ represents the leverage effect in portfolio returns, such a result might infer that the greater the leverage effect, the lower the Jensen's alpha.

Table 5: Posteriors Estimates of the System – First Set of Predictors

	cov (η, ε)	cov (u, ε)	cov (u, η)	α	ϕ	μ	γ_1	γ_2	γ_3	γ_4
All Cap Growth	0.0000	0.0000	-0.0003	0.0016	0.7127***	0.3419***	0.0054	0.0141	0.0124**	0.0048
All Cap Value	0.0000	0.0000	0.0000	0.0023***	0.6973***	0.2578	0.0023	0.0198	0.0053	0.0119
Energy Sector	0.0002	-0.0004***	-0.0003	0.0028	0.2464	0.7458***	0.0619	-0.0224	0.0511**	-0.1609**
Financials Sector	-0.0001	0.0002***	-0.0002	0.0051***	0.0621	0.7344***	-0.0467***	0.0165	0.0204***	0.1358***
Healthcare Sector	0.0002	-0.0003***	-0.0002	0.0024	0.3769	0.6685***	-0.0384	0.033	0.0676**	0.0336***
InfoTech Sector	0.0000	0.0006***	0.0000	0.0019	-0.0874	1.2791***	0.0715***	-0.053***	-0.0182***	-0.0397***
Large Cap Blend	0.0000	0.0000	-0.0001**	0.0014***	0.752***	0.2324***	0.0036***	0.002	0.003**	0.0038**
Large Cap Growth	0.0000	0.0000	0.0000	0.0017***	0.7583***	0.2504	0.0029	0.006	0.0048	0.0023
Large Cap Value	0.0000	0.0000	-0.0001	0.0027***	0.2061	0.6733***	-0.0172	0.03**	0.0132**	0.0274***
Materials Sector	0.0004	-0.0005***	-0.0009	0.0029	0.3577	0.5223***	0.1352**	0.0501	0.1501**	-0.1602***
Mid Cap Blend	0.0000	0.0000	-0.0003	0.0036***	0.3838	0.5167	-0.0028	-0.0315	0.0079	-0.0494
Mid Cap Growth	0.0000	0.0001	0.0000	0.0025**	0.0497	1.03***	-0.0112	0.0278**	0.017**	0.0433***
Mid Cap Value	0.0000	-0.0002***	-0.0001	0.0026***	0.8113***	0.1887***	0.0087***	0.013	0.0048	0.0053
Small Cap Blend	0.0000	-0.0002***	-0.0004	0.0026**	0.5341***	0.5042***	-0.0113	0.026	0.0376	0.0284***
Small Cap Growth	0.0000	0.0001	-0.0001	0.003**	0.3155	0.7447	-0.0295	0.0396	0.0331	0.0258
Small Cap Value	0.0001	-0.0003***	-0.0005	0.0026**	0.7662***	0.2467***	0.0149**	0.0219	0.018**	0.0094
Utilities Sector	0.0000	0.0001***	-0.0002	0.0039***	0.6721***	0.1313	-0.0038	-0.0248	-0.0289	0.0042
Mean	0.0001	0.0000	-0.0002	0.0027	0.4480	0.5335	0.0086	0.0099	0.0235	-0.0043
Min	-0.0002	-0.0006	-0.0010	0.0014	-0.0875	0.1314	-0.0467	-0.0530	-0.0289	-0.1609
Max	0.0005	0.0007	0.0001	0.0052	0.8114	1.2792	0.1353	0.0502	0.1502	0.1358
StdDev	0.0002	0.0003	0.0002	0.0009	0.2894	0.3180	0.0444	0.0278	0.0399	0.0704

The table reports estimates of the parameters of the system (3)-(6) for each EWP using the first set of predictors. For the system innovation distribution, the table reports the covariances. Other parameters are from equation (3) and (4). *, **, *** denote significance at 0.1, 0.5, and 0.01 levels respectively.

Inspection of beta dynamics gives various insights on how managers modify their risk exposure over time. Consider, first, the persistent parameter ϕ. The estimated values indicate average persistence in beta variation, though some differences occur for specific EWP depending on predictors. The mean coefficients are 0.448 and 0.3123 and take on values between

$\begin{bmatrix} -0.0875; 0.8114 \end{bmatrix}$ and $\begin{bmatrix} -0.0816; 0.7830 \end{bmatrix}$, for the first and second set. An in-depth analysis of data in Tables 5 and 6 also indicates that the set of predictors matter, at least for specific fund category. Note the InfoTech Sector, where ϕ is -0.0875 for the first set and takes on a value of 0.7829 for the second set, and the Mid Cap Value, in which ϕ ranges from 0.8113 to 0.1914. Interestingly, significant persistence coefficient tends to be high, especially when considering the first set of predictors, indicating weak mean-reversion and so high beta volatility. Funds with high ϕ then tend to deviate consistently and for a potentially long time from their average beta μ. This result is quite different from that of Mamaysky, Spiegel, and Zhang (2007) who instead found average persistence parameter between 0.12 and 0.35. Another interesting finding arises from Table 7. Indeed, we note that long-run beta and persistent parameter have significant negative correlation of -0.9472 and -0.8343, for the first and second set of predictors respectively. This indicates that the lower the long-run beta, the higher the persistence then reflecting in weak mean-reversion. This finding could suggest that dynamic funds with significant risk exposure variation should have low beta on average, since high persistence reflects high unconditional beta volatility.

A second major result is drawn from the beta sensitivities towards the instruments. Inspection of data in Tables 5 and 6 reveals that funds differ significantly in terms of instrument-based rules in beta variations. Supposing managers look at the first set of predictors in making estimates of expected benchmark returns before choosing the right risk exposure, the trend seems to be the most important predictor with the higher absolute average coefficient.[12] Also considering the statistical significance of the coefficient we note that for ten out of 17 EWPs we reach significance with positive sign for all of the ten EWPs except for the Info Tech Sector. The term spread is also a prominent instrument with nine significant coefficients while for default spread and dividend yield significance is reached for only six and three EWPs respectively. Of interest also is the relationship between Γ and Λ vectors, which can be seen by inspecting correlations between each element of Γ with the corresponding one of Λ. The correlations reported in Table 7 indicate that the relationship, if any, among coefficients is virtually non-existent: managers do not care about benchmark sensitivities in choosing their instrument exposure.

Supposing managers look at the second set of predictors in making estimates of expected benchmark returns, the term spread seems instead to be the most important predictor with higher absolute average coefficient[13], and with nine significant coefficients over 17. For inflation and illiquidity the statistical significance is reached for only five and one EWPs respectively. Also noteworthy is the relationship between the Γ and Λ vectors, which can be seen by inspecting correlations between each element of the vector Γ with the corresponding one of the vector Λ. The correlations in Table 7 reveal that managers might be aligned to benchmark sensitivity in

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[12] The absolute coefficient averages are 0.0275, 0.0254, 0.0290, 0.0439 for the first, second, third and fourth predictors.
[13] The absolute coefficient averages are 0.0084, 0.0308, 0.0420 for the first, second and third predictors.

choosing their exposure towards illiquidity. The correlation coefficient of 0.5353 is indeed positive and statistically significant at 0.05 level. Nevertheless, this result should be interpreted with care, since illiquidity reached significance for only one fund category.

Table 6: Posteriors Estimates of the System – Second Set of Predictors

	cov (η, ε)	cov (u, ε)	cov (u, η)	α	ϕ	μ	γ_1	γ_2	γ_3
All Cap Growth	0.0000	0.0000	-0.0006	0.0026**	0.3923***	0.6908***	-0.0021	0.0139	-0.0307***
All Cap Value	-0.0002	0.0000	-0.0005	0.0033***	0.1042	0.7318***	0.0054	0.0335***	0.0237**
Energy Sector	0.0006	-0.0003***	-0.0007	0.0036**	0.0479	0.9369***	-0.0231	0.0791	-0.1509***
Financials Sector	-0.0002	0.0002***	-0.0002	0.0051***	0.5065	0.3813	0.0166**	-0.0006	0.0631
Healthcare Sector	0.0001	-0.0004***	-0.0001	0.0028**	0.3895	0.6445***	-0.0013	0.0058	0.0138
InfoTech Sector	-0.0002	0.0002	-0.001	0.0032	0.7829***	0.259	-0.0143	-0.0345	-0.0406
Large Cap Blend	0.0000	0.0000	-0.0003***	0.0018***	0.336***	0.6068***	0.0037	0.013***	0.0102***
Large Cap Growth	0.0000	0.0000	-0.0005	0.0024***	0.5427***	0.4633***	0.0032	-0.0039	-0.0139
Large Cap Value	-0.0001	0.0000	-0.0002	0.0032***	-0.0815	0.8958***	0.0006	0.0385***	0.0359***
Materials Sector	0.0005	-0.0005***	-0.0011	0.004	0.1765	0.644***	-0.0299	0.0802	-0.1866***
Mid Cap Blend	-0.0001	0.0000	-0.0008**	0.0041***	0.2798	0.6028***	-0.007	0.0257	-0.0349***
Mid Cap Growth	0.0000	0.0000	-0.0005	0.0032***	0.4634**	0.565***	0.0087	0.0044	0.0065
Mid Cap Value	-0.0001	-0.0002***	-0.0008	0.004***	0.1914	0.7683***	-0.0049	0.0503**	-0.0087
Small Cap Blend	0.0000	-0.0002***	-0.0009**	0.0035***	0.2806	0.7436***	0.0051	0.0282	0.0131
Small Cap Growth	-0.0002	0.0000	-0.0012	0.0044***	0.3388	0.6862***	0.0095	0.0309	0.0072
Small Cap Value	0.0000	-0.0003***	-0.0016***	0.0046***	0.2072	0.7759***	-0.0069	0.0691**	-0.0346**
Utilities Sector	0.0000	0.0001***	0.0000	0.0032***	0.3504	0.2862***	0.0003	-0.0114	0.039***
Mean	0.0000	-0.0001	-0.0007	0.0035	0.3123	0.6284	-0.0021	0.0249	-0.0170
Min	-0.0002	-0.0005	-0.0016	0.0018	-0.0816	0.2590	-0.0300	-0.0345	-0.1867
Max	0.0006	0.0002	-0.0001	0.0052	0.7830	0.9370	0.0167	0.0803	0.0631
StdDev	0.0003	0.0002	0.0004	0.0008	0.2038	0.1915	0.0118	0.0319	0.0642

The table reports estimates of the parameters of the system (3)-(6) for each EWP using the second set of predictors. As for the system innovation distribution, the table reports covariances among shocks. Other parameters are from equation (3) and (4). *, **, *** denote significance at 0.1, 0.5, and 0.01 levels respectively.

Table 7: Parameter Correlations

	corr (η, ε)	corr (u, ε)	corr (u, η)	α	ϕ	μ	γ_1	γ_2	γ_3	γ_4
				Panel A: Parameter Correlations – First Set of Predictors						
corr (u, ε)	**-0.6538**									
corr (u, η)	-0.0143	**0.5677**								
α	**-0.5247**	0.3278	-0.0711							
ϕ	0.2124	**-0.5123**	**-0.5122**	-0.3524						
μ	-0.0017	0.4265	**0.5847**	0.0949	**-0.9472**					
γ_1	0.3696	-0.2208	0.1405	-0.2407	-0.0940	0.1203				
γ_2	0.0202	-0.3617	-0.2237	-0.0291	0.1375	-0.1573	-0.1727			
γ_3	0.3394	-0.4676	-0.1250	0.0490	-0.1641	0.1160	**0.4892**	**0.5839**		
γ_4	-0.4563	0.3577	0.0049	0.2344	0.0114	-0.0325	**-0.8638**	0.2437	**-0.4872**	
λ_1	**0.5665**	**-0.7359**	-0.4831	-0.1859	**0.6032**	**-0.5411**	0.2430	0.1406	0.1507	-0.2730
λ_2	-0.4549	**0.6370**	0.1334	0.0653	-0.2633	0.1843	-0.4130	-0.0346	-0.1953	0.3231
λ_3	0.3439	**-0.5701**	**-0.5314**	**-0.5723**	0.3820	-0.1989	-0.1951	0.1311	-0.1437	0.1054
λ_4	0.3445	**-0.5503**	-0.3558	-0.2295	0.3957	-0.3294	**0.5169**	-0.0010	0.0826	-0.4427
	corr (η, ε)	corr (u, ε)	corr (u, η)	α	ϕ	μ	γ_1	γ_2	γ_3	
				Panel B: Parameter Correlations – Second Set of Predictors						
corr (u, ε)	**-0.5224**									
corr (u, η)	0.0432	0.2877								
α	-0.1664	-0.0570	-0.0698							
ϕ	0.2408	0.3000	0.0276	-0.1546						
μ	-0.0353	**-0.5755**	-0.3076	0.0837	**-0.8343**					
γ_1	-0.4459	0.4831	0.0755	-0.0181	0.2294	-0.2088				
γ_2	0.1296	**-0.6147**	-0.2876	0.3872	**-0.8013**	**0.8077**	-0.5116			
γ_3	**-0.5540**	0.4890	0.2159	-0.0841	0.2053	-0.2790	**0.9128**	**-0.6075**		
λ_1	-0.2386	0.3323	0.2682	-0.4465	0.1356	-0.1559	**0.5353**	**-0.5093**	**0.5811**	
λ_2	-0.2821	**0.6211**	-0.0156	0.2046	0.3806	**-0.5212**	0.1735	-0.3986	0.2235	
λ_3	-0.1372	-0.1765	-0.4389	-0.1917	-0.1876	0.1444	0.0535	0.1188	0.1545	

This table reports correlations among parameters of the 17 EWPs. Panels A and B show correlations when the first and the second set of instruments are used respectively. Numbers in bold font are significant at 0.05 level.

VI. Betas and Market Timing

Within our framework, a manager is a market timer if $\mathrm{corr}(u, \eta)$ is positive. As in Becker, et al. (1999), we distinguish timing ability that merely reflects publicly available information, as captured by the set of instrumental variables, from conditional market timing based on better information. But, unlike those authors, we also consider imperfect predictability and a stochastic component in the process for systematic risk. Table 8 reports correlations among shocks in the system, giving information on both Bayesian conditional market timing ability measured by

$\text{corr}(u, \eta)$ and leverage effect measured by $\text{corr}(\eta, \varepsilon)$. Furthermore, the table also reports the correlation between portfolio return innovations and benchmark innovations, $\text{corr}(u, \varepsilon)$. Note that, as discussed in Section II.2, portfolio innovations are functions of beta innovations, and so $\text{corr}(u, \varepsilon)$ should depend on the correlation between beta and benchmark innovations.

Table 8: Shocks Correlations

	Panel A: Shocks Correlations					
	First Set of Predictors			Second Set of Predictors		
	corr (η, ε)	corr (u, ε)	corr (u, η)	corr (η, ε)	corr (u, ε)	corr (u, η)
All Cap Growth	-0.0033	-0.0755	-0.3176***	0.02	-0.0873	-0.3603***
All Cap Value	-0.0715	0.0883	-0.1187	-0.4151***	0.1099	-0.4214***
Energy Sector	0.1079	-0.2289***	-0.1834***	0.2162***	-0.1885***	-0.2988***
Financials Sector	-0.2344***	0.2613***	-0.1925***	-0.1957***	0.2218***	-0.1526***
Healthcare Sector	0.1979***	-0.2025***	-0.1506***	0.0954	-0.2881***	-0.1392**
InfoTech Sector	0.0623	0.3141***	0.1173	-0.0593	0.1088	-0.4012***
Large Cap Blend	0.0563	-0.1698***	-0.4461***	0.0171	0.0051	-0.4369***
Large Cap Growth	-0.0162	0.0819	-0.0444	-0.0643	0.0428	-0.4048***
Large Cap Value	-0.1699***	0.0779	-0.274***	-0.4362***	0.1168	-0.2403***
Materials Sector	0.0717	-0.2021***	-0.2463***	0.0983	-0.2003***	-0.3552***
Mid Cap Blend	-0.0704	0.0363	-0.3832***	-0.2082***	0.0384	-0.5194***
Mid Cap Growth	0.015	0.0975	-0.0032	-0.0617	0.0613	-0.3817***
Mid Cap Value	0.055	-0.3211***	-0.2171***	-0.1677***	-0.2804***	-0.41***
Small Cap Blend	0.0469	-0.19***	-0.3936***	0.0457	-0.2013***	-0.5161***
Small Cap Growth	0.0048	0.091	-0.203***	-0.074	0.0612	-0.4825***
Small Cap Value	0.1267**	-0.3273***	-0.4359***	-0.0433	-0.3001***	-0.5222***
Utilities Sector	-0.034	0.203***	-0.221***	-0.0269	0.2113***	-0.0664
Mean	0.0085	-0.0274	-0.2185	-0.0741	-0.0334	-0.3594
Min	-0.2344	-0.3273	-0.4462	-0.4363	-0.3001	-0.5222
Max	0.1980	0.3141	0.1174	0.2163	0.2218	-0.0665
StdDev	0.1057	0.2016	0.1535	0.1708	0.1766	0.1377
	Panel B: Hypothesis Testing for Correlation Equivalence					
	corr $(\eta, \varepsilon)_1$ minus corr $(\eta, \varepsilon)_2$		corr $(\upsilon, \varepsilon)_1$ minus corr $(\upsilon, \varepsilon)_2$		corr $(u, \eta)_1$ minus corr $(u, \eta)_2$	
Mean	0.0827		0.006		0.1409	
t-stat	2.866**		0.321		3.2666***	

The table shows the correlations among shocks in our system for each EWP. Panel A reports correlations computed by using the first and second set of predictors. Panel B reports the t-test for the hypothesis of correlation equivalence between shocks when the two sets of predictors are used. *, **, *** denote significance at 0.1, 0.5, and 0.01 levels respectively.

In Table 8 we report the correlations among shocks in our system computed for each EWP. Panel A shows correlations when using the first and second set of predictors and Panel B reports the t-test for the hypothesis of correlation equivalence between shocks when the two sets of predictors

are used. Consider, first, $\mathrm{corr}(u, \eta)$, which gives indications on conditional market timing ability of mutual funds. Both considering the first and the second set of predictors, no mutual fund category proves to be a significant market timer over the period 1990-2005. Indeed, no correlation appears to be positive and statistically significant. We note however that only the Info Tech Sector exhibits a positive correlation of 0.1173 which is very near to significance at 0.1 level (the p-value is indeed around 0.105) and may lead us to consider the Info Tech Sector as a "persistent" market timer over the time period inspected. Interestingly, this is the unique fund category for which we detect some conditional market timing. Again, we note also that instruments significantly affect the timing ability assessment: Panel B of the table shows that the second set of predictors leads to stronger average negative correlations between shocks u and η.

The leverage effect measured by $\mathrm{corr}(\eta, \varepsilon)$ gives further interesting results. While most consider such an effect to play a minor, if any, role, our results show that for some fund categories the leverage effect still matters. In Table 8 we observe that when considering the first set of predictors the Healthcare Sector and the Small Cap Value exhibit a significant positive correlation, while for Financials Sector and Large Cap Value the coefficient is negative and statistically significant. On the other hand, when using the second set of predictors, Energy Sector is the unique category for which the correlation is significantly positive, while All Cap Value, Financial Sectors, Large Cap Value, Mid Cap Blend and Mid Cap Value exhibit a significant negative correlation. Although of less magnitude, Panel B shows that, also in this case, instruments significantly affect the leverage effect assessment: the second set of predictors leads in fact to a reduction in the average correlation between shocks η and ε. Since on average we had negative conditional market timing, the negative leverage effect is, in a sense, good for mutual funds: anomalous negative timing effect is indeed reduced by innovations in portfolio returns. And again, negative leverage effect induces a suitable positive correlation between portfolio and benchmark innovations. This is easy to understand if we assume a structure among innovations as described in Section II.2, namely η and ε to be functions of, respectively, u and η. In this way, $\mathrm{corr}(\eta, \varepsilon)$ would affect $\mathrm{corr}(u, \varepsilon)$. Such reasoning leads one to expect $\mathrm{corr}(u, \varepsilon)$ and $\mathrm{corr}(\eta, \varepsilon)$ moving in opposite directions. And this is what we found by computing correlations between the two parameters. Indeed, Table 7 shows that in both the two set of predictors $\mathrm{corr}[\mathrm{corr}(u, \varepsilon), \mathrm{corr}(\eta, \varepsilon)]$ is significantly negative. Again in Table 7 Panel A, interestingly we note that the correlation between the leverage and the Jensen's alpha is significantly negative. So, it seems that the negative leverage effect not only makes offsetting negative market timing possible, but also leads to an increment in the extra performance of mutual funds, i.e. selectivity. However, this finding holds for the first set of predictors only.

VII. Beta Decomposition

Another major point examined in our study regards the variance decomposition of beta. This is because we are interested in the contribution of persistence, instruments and shocks in explaining the beta dynamics. On the issue of conditioning information impact on betas, Mamaysky, et al. (2007) did not find any significance in adding Treasury bill and dividend yield on the CRSP equally-weighted index in their Kalman process; the estimated coefficients were statistically indistinguishable from zero thus leading them to conclude that few funds use macroeconomic variables, indeed most do not. This is precisely our concern: the instrument impact on beta dynamics. More broadly, we want to see what the prominent factor in time-varying betas is, in terms of explained variance. As is obvious, to do this we focus on equation (4). More precisely, we decompose the variance of beta according to the classical variance decomposition by using coefficient estimates and covariances of all the explanatory variables which enter into equation (4). Since the explanatory variables are not orthogonal, in computing and decomposing the total variation in beta we would necessarily consider covariances terms. But because the main interest is the role of each beta component, we can bypass this problem by focusing on the three main factors: the mean reverting term, the instruments viewed as a whole, and the shocks. In this way, the percent variance contribution of the factors is computed as follows. Let $var_1 = \phi^2 \, var\left(L\beta_{p,t}\right)$ be the variance explained by the mean reverting beta term, $var_2 = \Gamma' covar\left(\mathbf{z}_{t-1}\right)\Gamma$ be the variance explained by the instruments, and $var_3 = var\left(\eta_{p,t}\right)$ be the variance explained by the shocks, then

$$v_1 = \frac{var_1}{var_1 + var_2 + var_3}; \; v_2 = \frac{var_2}{var_1 + var_2 + var_3}; \; v_1 = \frac{var_3}{var_1 + var_2 + var_3}$$

are the per cent contributions of each factor in beta variation (excluding covariance terms). Table 9 reports results of variance decompositions relative to each EWP for the two sets of predictors. On average, lagged betas play a key role in explaining beta variation, for both sets of instruments. However, specific fund styles tend to behave very differently. In the first set of predictors the cross-section average v_1 is near 0.56, but we also observe that Mid Cap Value accounts for only 6.08 per cent, while Info Tech Sector for 99.22 per cent, i.e. virtually everything is explained by the lagged beta. When using the second set of predictors, while on the one hand we obtain a similar cross-section v_1 of about 0.58 and similar min-max values, on the other we observe that Info Tech Sector accounts for 5.1 per cent. It is then clear that instruments still matter, at least for some fund styles. As regards the importance of instruments in explaining the beta variation, our results are different from those of Mamaysky, et. al (2007). Table 9 shows that the averages of v_2 are 42.27 and 18.5 per cent, for the first and second set respectively. And again, min-max values range for both sets from 0 to above 90 per cent. Instrument selection matters also in this case, since when changing conditioning variables some funds exhibit a reversal in magnitude, as

is the case of Info Tech Sector and Mid Cap Value, for which the corresponding ranges are 0.61–90.2 and 93.62–3.5 per cent passing from the first to the second set of predictors.

Table 9: Variance Decomposition of Betas

	First Set of Predictors			Second Set of Predictors		
	Lagged Betas	Instruments	Shocks	Lagged Betas	Instruments	Shocks
All Cap Growth	0.1781	0.7663	0.0557	0.5540	0.2600	0.1860
All Cap Value	0.2271	0.7549	0.0180	0.7650	0.0110	0.2240
Energy Sector	0.9090	0.0775	0.0135	0.9690	0.0020	0.0290
Financials Sector	0.9894	0.0040	0.0065	0.5900	0.3390	0.0700
Healthcare Sector	0.7605	0.2212	0.0183	0.5780	0.1170	0.3050
InfoTech Sector	0.9922	0.0061	0.0017	0.0510	0.9020	0.0470
Large Cap Blend	0.1046	0.8294	0.0660	0.5490	0.1320	0.3190
Large Cap Growth	0.1533	0.7999	0.0468	0.2480	0.4870	0.2640
Large Cap Value	0.9317	0.0551	0.0132	0.9380	0.0050	0.0570
Materials Sector	0.7872	0.1934	0.0194	0.9250	0.0410	0.0350
Mid Cap Blend	0.7577	0.2038	0.0385	0.6330	0.1050	0.2620
Mid Cap Growth	0.9880	0.0023	0.0097	0.1170	0.2370	0.6450
Mid Cap Value	0.0608	0.9362	0.0030	0.7120	0.0350	0.2530
Small Cap Blend	0.4998	0.4609	0.0393	0.4990	0.0730	0.4280
Small Cap Growth	0.8197	0.0958	0.0845	0.3700	0.1080	0.5220
Small Cap Value	0.0966	0.8814	0.0220	0.6730	0.0440	0.2830
Utilities Sector	0.2578	0.7273	0.0150	0.7380	0.2390	0.0220
Mean	0.5596	0.4127	0.0277	0.5830	0.1850	0.2320
Min	0.0608	0.0023	0.0017	0.0510	0.0020	0.0220
Max	0.9922	0.9362	0.0845	0.9690	0.9020	0.6450
StdDev	0.3709	0.3649	0.0237	0.2670	0.2280	0.1820

This table reports the variation in betas according to the equation (4) for each EWP. The values are computed without considering covariances among explanatory variables in (4). Lagged betas, instruments (first and second set) and shocks decompose the total beta variance summing to unity since no covariances terms are considered.

Finally, the stochastic beta variation component plays a very minor role for the first set of predictors, while for the second set of predictors average weight is large and significant: 23.30 per cent. Therefore fund managers seem to use the default spread, the dividend yield, the term spread, and the S&P index trend in altering their portfolio allocations, as expressed by beta variations. Such a conclusion can be easily tested by computing $\mathrm{corr}\left[\beta, E(r_{m,t})\right]$. Table 10 reports the results which indicate that betas are strongly correlated to the benchmark expectation only when the first set of predictors are used in making, first, market forecasts, and, second, beta variation.

Table 10: Beta Correlations

	$\mathrm{corr}\left[\beta, E\left(r_{m,t}\right)\right]$ using the First Set of Predictors	$\mathrm{corr}\left[\beta, E\left(r_{m,t}\right)\right]$ using the Second Set of Predictors
All Cap Growth	**0.8690**	-0.1701
All Cap Value	**0.8328**	-0.2680
Energy Sector	**-0.6764**	-0.4556
Financials Sector	0.4724	0.1933
Healthcare Sector	**0.5796**	-0.0956
InfoTech Sector	**-0.8183**	-0.2277
Large Cap Blend	**0.6476**	-0.0829
Large Cap Growth	**0.6362**	0.0660
Large Cap Value	**0.9080**	-0.3258
Materials Sector	-0.3152	-0.4193
Mid Cap Blend	**-0.7522**	-0.3529
Mid Cap Growth	**0.8169**	0.1213
Mid Cap Value	**0.9323**	**-0.5751**
Small Cap Blend	**0.7736**	-0.2521
Small Cap Growth	**0.8045**	-0.1797
Small Cap Value	**0.9100**	**-0.5455**
Utilities Sector	-0.4260	0.1354
Mean	0.3644	-0.2020
Min	-0.8183	-0.5751
Max	0.9323	0.1933
StdDev	0.6606	0.2355

This table reports the correlations between the beta of each EWP estimated according to the equation (4) and the benchmark expectation as in equation (5), i.e. considering $E(r_{m,t}) = \Lambda' \mathbf{z}_{t-1}$. The correlations are computed when the first and the second set of instruments are used respectively. Numbers in bold font are significant at 0.05 level.

VIII. Conclusion

Although empirical evidence proved some stock return predictability using conditioning variables, it is not clear how such predictability translates into portfolio rebalancing. Suppose a fund manager first looks at predictors making estimates of future benchmark returns, then alters the portfolio allocation. How does the manager use predictors in changing her/his portfolio structure over time? This is what we analyse in this paper. To do this we derived a new model within a Bayesian framework where managers are assumed to modulate the systematic risk in part by observing how the benchmark returns are related to a set of predictors, and in part on the basis of their own informational set, which is stochastically inaccessible for econometricians who basically observe the return patterns over time. In doing so, managers take into account the potential benefit arising from the market timing exploitable by the benchmark predictors and by the private information.

To translate this view into a formal model we introduced an approach which combines time-variation, a stochastic component, and a deterministic component in the beta process, using a predictive system given by the portfolio excess return, the time-varying beta and benchmark excess return equations. Predictors are assumed to be imperfect and innovations in the system are correlated according to some data-theoretical priors. In this way, we inspect how managers really use predictors in changing investments over time. Furthermore, the system also delivers a measure for conditional market timing which is different from that of traditional conditional asset pricing models, because we accommodate imperfect predictors and beliefs about correlations among innovations.

The empirical study proved that instruments impact significantly on beta dynamics, but managers do not care about benchmark sensitivities towards the predictors in choosing their instrument exposure, both in sign and in magnitude. The main reason is the modest benchmark forecasting power of such instruments. Persistence in beta is significant although we noted strong differences among fund categories. Interestingly, long-run beta and persistent parameter are negatively correlated. This signifies that weak mean-reversion mutual funds show low long-run beta with significant beta variation. According to a common finding on market timing we did not find evidence of timing ability. However, we found a significant leverage effect. And when the market timing is anomalously negative, the negative leverage effect leads to positively correlating portfolio and benchmark innovations, thus offsetting the negative timing effect.

Appendix A: conditional posterior distributions
A.1 Posterior simulation of the betas

Conditioned on all parameters and all the data on the benchmark $(r_{mt}, t = 1,2,..T)$, we *de facto* observe u_{mt} and therefore

(17)
$$\begin{pmatrix} \varepsilon_{pt} \\ \eta_{pt} \end{pmatrix} | u_{mt} \sim NID\left[\begin{pmatrix} \mu_{1t} \\ \mu_{2t} \end{pmatrix}, \Sigma_{12|3} \right],$$

$$\Sigma_{12|3} = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix} - \begin{bmatrix} \sigma_{\varepsilon u} \\ \sigma_{\eta u} \end{bmatrix} \sigma_u^{-2} \begin{bmatrix} \sigma_{\varepsilon u} & \sigma_{\eta u} \end{bmatrix},$$

$$\begin{pmatrix} \mu_{1t} \\ \mu_{2t} \end{pmatrix} = \begin{bmatrix} \sigma_{\varepsilon u} \\ \sigma_{\eta u} \end{bmatrix} \sigma_u^{-2} u_{mt}.$$

This of course will have an impact on the errors and the intercepts of equations (1) and (2) and the resulting KF + CK procedure. Conditional on r_{mt} and on the parameters in equation (4), the state space becomes

(18)
$$r_{pt} = \alpha_p + \mu_{1t} + \beta_t r_{mt} + \varepsilon_{pt}^*,$$

$$\beta_t = c + \mu_{2t} + \phi\beta_{t-1} + \Gamma' \mathbf{z}_{t-1} + \eta_{pt}^*,$$

$$\begin{bmatrix} \varepsilon_{pt}^* \\ \eta_{pt}^* \end{bmatrix} \sim NID\left[\begin{pmatrix} \mu_{1t} \\ \mu_{2t} \end{pmatrix}, \Sigma_{12\beta} \right].$$

We can run the Kalman filter on this system via repetition of the following steps:

- Initialisation (at *t=0*): $q_{0|0}$ and $\beta_{0|0}$ (using unconditional distribution of β)
  At each *t=0,1,…,T-1,* the projection step:
  $$\beta_{t+1|t} = c + \mu_{2t} + \phi\beta_{t|t} + \Gamma' \mathbf{z}_t$$
  $$\beta_{t+1} - \beta_{t+1|t} = \phi(\beta_t - \beta_{t|t}) + \eta_{pt+1}^* = \eta_{pt+1|t}^*$$
  $$V(\varepsilon_{pt+1|t}^* | I_t) = q_{t+1|t} = \phi^2 q_{t|t} + \sigma_{\eta\eta}^*$$
  and

$$r_{pt+1} = r_{p+1|t} + \varepsilon^*_{pt+1|t},$$

$$\varepsilon^*_{pt+1|t} = r_{mt+1}\varepsilon^*_{pt+1|t} + \varepsilon^*_{pt+1},$$

$$V(\varepsilon^*_{pt+1|t}|I_t) = r^2_{mt+1}q_{t+1|t} + \sigma^*_{\varepsilon\varepsilon} + 2r_{mt+1}\sigma^*_{\varepsilon\eta} = \sigma^*_{\varepsilon\varepsilon,t+1|t},$$

$$Cov(\varepsilon^*_{pt+1|t},\eta^*_{2t+1|t}|I_t) = r_{mt+1}q_{t+1|t} + \sigma^*_{\varepsilon\eta} = \sigma^*_{\varepsilon\eta,t+1|t}.$$

- At each t=0,1,…,T-1, the update step

$$\beta_{t+1|t+1} = \beta_{t+1|t} + \frac{\sigma^*_{\varepsilon\eta,t+1|t}}{\sigma^*_{\varepsilon\varepsilon,t+1|t}}\varepsilon^*_{pt+1|t}$$

$$q_{t+1|t+1} = q_{t+1|t} - \frac{\sigma^*_{\varepsilon\eta,t+1|t}}{\sigma^*_{\varepsilon\varepsilon,t+1|t}}.$$

Exploiting the Markov property of the system, the conditional distributions obtained with the Kalman filter are then used, following Carter and Kohn (1994) to obtain a draw from the posterior distribution of $\beta_t, t = 0,1,..,T$, conditional on all parameters and all data evidence:

$$p(\beta_0,\beta_1,...,\beta_T|\theta,\mathbf{r}_{pT},\mathbf{r}_{mT}) = p(\beta_T|\theta,\mathbf{r}_{pT},\mathbf{r}_{mT})\prod_{t=0}^{T-1}p(\beta_t|\beta_{t+1},\theta,\mathbf{r}_{pt},\mathbf{r}_{mt})$$

(19)
$$p(\beta_t|\beta_{t+1},\theta,\mathbf{r}_{pt},\mathbf{r}_{mt}) = N(\beta_{t|t+1},q_{t|t+1})$$

$$\beta_{t|t+1} = \beta_{t|t} + \frac{\phi q_{t|t}}{q_{t+1|t}}(\beta_{t+1} - \beta_{t+1|t}), q_{t|t+1} = q_{t|t} - \frac{(\phi q_{t|t})^2}{q_{t+1|t}}.$$

A.2 Posterior simulation of α_p

Conditioned on all other parameters ($\bar{\theta}_\alpha$), on the series β_T and the data, we observe the shocks η_{pt} and u_{mt}. Therefore the conditional distribution of ε_{pt} is

$$\left(\varepsilon_{pt}\left|\begin{bmatrix}\eta_{pt}\\u_{mt}\end{bmatrix}\right.\right) \sim NID\left[(\mu_{1t}),\sigma^{*2}_\varepsilon\right]$$

(20)
$$\sigma^{*2}_\varepsilon = \sigma^2_\eta - \begin{bmatrix}\sigma_{\varepsilon\eta} & \sigma_{\varepsilon u}\end{bmatrix}\begin{bmatrix}\sigma_{\eta\eta} & \sigma_{\eta u}\\\sigma_{\eta u} & \sigma_{uu}\end{bmatrix}^{-1}\begin{bmatrix}\sigma_{\varepsilon\eta}\\\sigma_{\varepsilon u}\end{bmatrix}$$

$$\mu_{1t} = \begin{bmatrix}\sigma_{\varepsilon\eta} & \sigma_{\varepsilon u}\end{bmatrix}\begin{bmatrix}\sigma_{\eta\eta} & \sigma_{\eta u}\\\sigma_{\eta u} & \sigma_{uu}\end{bmatrix}^{-1}\begin{bmatrix}\eta_{pt}\\u_{mt}\end{bmatrix}.$$

This of course will change the intercept in the first equation:

$$
\begin{aligned}
y_{1t} &= r_{mt} - \mu_{1t} - \beta_t r_{mt} = \alpha_p + \varepsilon_{pt}^*, \\
\varepsilon_{pt}^* &\sim N\!\left(0, \sigma_{1/2,3}^2\right).
\end{aligned}
$$

(21)

This result, together with a Gaussian prior pdf for α_p, with moments $\underline{\mu}_\alpha, \underline{\sigma}_\alpha^2$ produces a conditional posterior which is Gaussian:

$$
p(\alpha_p \mid \overline{\boldsymbol{\theta}}_\alpha, \boldsymbol{\beta}_t \mathbf{r}_{mt}, \mathbf{r}_{pt}) = N(\overline{\mu}_a, \overline{\sigma}_\alpha^2)
$$

(22)

$$
\overline{\mu}_a = \frac{\displaystyle\sum_{t=1}^{T} y_{1t}}{\sigma_\varepsilon^{*2}} + \frac{\underline{\mu}_\alpha}{\underline{\sigma}_\alpha^2}, \quad \overline{\sigma}_\alpha^2 = \left(\frac{1}{\underline{\sigma}_\alpha^2} + \frac{T}{\sigma_\varepsilon^{*2}}\right)^{-1}.
$$

A.3 Posterior simulation of the parameters in the beta equation

The parameters to be drawn are $\boldsymbol{\theta}_\beta = [c, \phi, \Gamma']$. Conditioning on the series of the β_ts and all the other parameters of the model ($\overline{\boldsymbol{\theta}}_\beta$), we then observe the whole sequence of ε_{pt} and u_{mt}. Therefore, the conditional distribution of η_{pt} becomes

$$
\left(\eta_{pt} \left\| \begin{bmatrix} \varepsilon_{pt} \\ u_{mt} \end{bmatrix} \right.\right) \sim NID\!\left[(\mu_{2t}), \sigma_{\eta|\varepsilon u}^{*2}\right]
$$

(23)

$$
\sigma_{\eta|}^{*2} = \sigma_\eta^2 - \begin{bmatrix} \sigma_{\eta\varepsilon} & \sigma_{\eta u} \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon u} \\ \sigma_{\varepsilon u} & \sigma_u^2 \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{\eta\varepsilon} \\ \sigma_{\eta u} \end{bmatrix}
$$

$$
\mu_{2t} = \begin{bmatrix} \sigma_{\eta\varepsilon} & \sigma_{\eta u} \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon u} \\ \sigma_{\varepsilon u} & \sigma_u^2 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{pt} \\ u_{mt} \end{bmatrix}.
$$

This induces an extra intercept term in equation (4) which is accounted for by defining as dependent variable and regressors in this equation:

$$
\begin{aligned}
\mathbf{y}_2 &= \mathbf{X}_\beta \boldsymbol{\theta}_\beta + \boldsymbol{\eta}^*, \\
y_{2t} &= \beta_t - \mu_{1t}, \mathbf{x}_{t,\beta} = [1, \beta_{t-1}, \Gamma'].
\end{aligned}
$$

(24)

Therefore, using for θ_β a Gaussian prior distribution with moments $\underline{\mu}_{\theta_\beta}$ and $\underline{\Sigma}_{\theta_\beta}$, we can apply the usual conditional conjugate results and obtain a conditional posterior distribution for θ_β which is Gaussian:

(25)
$$p(\theta_\beta \mid \overline{\theta}_{\theta_\beta}, \beta_t, \mathbf{r}_{mt}, \mathbf{r}_{pt}) = N(\vec{\mu}_{\theta_\beta}, \overline{\Sigma}_{\theta_\beta})$$

$$\vec{\mu}_{\theta_\beta} = \frac{\mathbf{X}'_\beta \mathbf{y}_1}{\sigma_{\eta l}^{*2}} + \underline{\Sigma}_{\theta_\beta}^{-1} \underline{\mu}_{\theta_\beta}, \quad \overline{\Sigma}_{\theta_\beta} = \left(\frac{\mathbf{X}'_\beta \mathbf{X}_\beta}{\sigma_{\eta l}^{*2}} + \underline{\Sigma}_{\theta_\beta}^{-1} \right)^{-1}.$$

A.4 Posterior simulation of the Λ parameters

Conditioning on the β_t sequence and all the other parameters of the model ($\overline{\theta}_\Lambda$), then it is as if we observe ε_{pt} and η_{pt}. Therefore the conditional distribution of u_{mt} is:

(26)
$$\left(u_{mt} \left\| \begin{bmatrix} \varepsilon_{pt} \\ \eta_{pt} \end{bmatrix} \right) \sim NID\left[(\mu_{3t}), \sigma_u^{*2} \right]$$

$$\sigma_u^{*2} = \sigma_u^2 - \begin{bmatrix} \sigma_{u\varepsilon} & \sigma_{u\eta} \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{u\varepsilon} \\ \sigma_{u\eta} \end{bmatrix}$$

$$\mu_{2t} = \begin{bmatrix} \sigma_{u\varepsilon} & \sigma_{u\eta} \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{pt} \\ \eta_{pt} \end{bmatrix}.$$

this induces an extra intercept term in equation (5) which will be then accounted for by defining the dependent variable and regressors of this equation:

(27)
$$\mathbf{y}_3 = \mathbf{Z}_{-1}\Lambda + \mathbf{u}^*,$$
$$y_{3t} = r_{mt} - \mu_{3t}.$$

Therefore, using for Λ a Gaussian prior with moments $\underline{\mu}_\Lambda$ and $\underline{\Sigma}_\Lambda$, we can apply the usual conditional conjugate results and obtain a conditional posterior distribution for Λ which is Gaussian:

$$p(\,\boldsymbol{\Lambda}\,/\overline{\boldsymbol{\theta}}_{\Lambda}\,,\boldsymbol{\beta}_{T}\,,\mathbf{r}_{mt}\,,\mathbf{r}_{pt}\,)=N(\,\bar{\boldsymbol{\mu}}_{\Lambda}\,,\overline{\boldsymbol{\Sigma}}_{\Lambda}\,)$$

(28)
$$\bar{\boldsymbol{\mu}}_{\theta_{\beta}}=\frac{\mathbf{Z}_{-1}^{'}\mathbf{y}_{3}}{\sigma_{\upsilon}^{*2}}+\underline{\boldsymbol{\Sigma}}_{\Lambda}^{-1}\underline{\boldsymbol{\mu}}_{\Lambda}\,,\overline{\boldsymbol{\Sigma}}_{\Lambda}=\left(\frac{\mathbf{Z}_{-1}^{'}\mathbf{Z}_{-1}}{\sigma_{u}^{*2}}+\underline{\boldsymbol{\Sigma}}_{\Lambda}^{-1}\right)^{-1}.$$

A.5 Posterior simulation of the Σ parameters

Conditional on the data, the betas and the remaining parameters ($\overline{\boldsymbol{\theta}}_{\Sigma}$), using a Wishart prior for $\boldsymbol{\Sigma}^{-1}$, we obtain, via the usual conjugate results, a Wishart conditional posterior:

(29)
$$p\!\left(\boldsymbol{\Sigma}^{-1}/\overline{\boldsymbol{\theta}}_{\Sigma}\,,\boldsymbol{\beta}_{T}\,,\mathbf{r}_{mt}\,,\mathbf{r}_{pt}\,\right)=W_{n}\!\left(\mathbf{H}\,/\overline{\nu}\,,\overline{\mathbf{S}}\right),$$
$$\overline{\nu}=T+\underline{\nu}\,,\overline{\mathbf{S}}=\mathbf{S}+\underline{\mathbf{S}},$$
$$\mathbf{S}=\sum_{t=1}^{T}\boldsymbol{\eta}_{t}\boldsymbol{\eta}_{t}^{'}.$$

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